A free boundary problem in thermal insulation with a prescribed heat source

Paolo Acampora, Emanuele Cristoforoni

A joint work with C. Nitsch, and C. Trombetti

Abstract

We study the thermal insulation of a bounded body $\Omega \subset \mathbb{R}^n$, under a prescribed heat source $f > 0$, via a bulk layer of insulating material. We consider a model of heat transfer between the insulated body and the environment determined by convection; this corresponds to Robin boundary conditions on the free boundary of the layer. We show that a minimal configuration exists and that it satisfies uniform density estimates.

On the symmetric rearrangement of the gradient of a Sobolev function

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In this poster we generalize a classical comparison result for solutions to Hamilton-Jacobi equations with Dirichlet boundary conditions, to solutions to H-J equations with non zero boundary trace.

As a consequence, we prove the isoperimetric inequality for the torsional rigidity (with Robin B.C.) and for other functionals involving such boundary conditions.

This is a joint work with Andrea Gentile.

Shape optimization for a nonlinear elliptic problem related to thermal insulation

Rosa Barbato

In this paper we consider a minimization problem of the type

$$I_{\beta,p}(D;\Omega) = \inf\left\{\int_\Omega |D\phi|^p dx + \beta\int_{\partial^*\Omega} |\phi|^p d\mathcal{H}^{n-1},\ \phi \in W^{1,p}(\Omega),\ \phi \geq 1 \text{ in } D\right\}$$

for some positive constant β, where $D \subset \bar{\Omega}$ is a compact connected set.
We let the set D vary, under prescribed geometrical constraints and $\Omega \setminus D$ of fixed thickness to look for the best (or worst) geometry in terms of maximization (or minimization) of $I_{\beta,p}$.
We restrict our analysis to the planar case and to the n-dimensional case, for convex sets.

Segregated solutions for some nonlinear Schrödinger systems with critical growth

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I will focus on the existence and multiplicity of positive non-radial solutions for some systems of Schrödinger equations in a weak fully attractive or repulsive regime in presence or in absence of an external radial trapping. The results have been obtained in collaboration with Maria Medina (Universidad Autonoma de Madrid), Angela Pistoia (La Sapienza Università di Roma) and Giusi Vaira (Università di Bari).

References

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On the number of critical points of solutions of elliptic problems

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A classical problem in partial differential equations concerns the study of classical solution of the problem

$$-\Delta u = f(u) \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega,$$

where $\Omega \subseteq \mathbb{R}^N$, $N \geq 2$ is a smooth and bounded domain and f a smooth nonlinearity. In particular we want to focus on the number of critical points of u and to investigate the role of the convexity of the domain Ω. If u is positive we discuss some generalization of existing results involving the sign of the curvature of $\partial\Omega$ [1, 2]. Finally if u is a sign-changing solution one can prove that the second eigenfunction of the Laplacian admits exactly two critical points if Ω is a planar and convex domain with large eccentricity [3].

References

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Long time behavior and stability for the surface diffusione flow in dimensions greater than three

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We consider the evolution in time of smooth subsets E_t of the Euclidean space such that their boundaries ∂E_t, which are smooth hypersurfaces, move, for $t \in [0, T)$, with "outer" normal velocity given by the Laplacian of the mean curvature of ∂E_t. The resulting motion is then called *surface diffusion flow*, proposed by Mullins in [6] to study thermal grooving in material sciences and first analyzed mathematically more in detail in [4]. In particular, in the physically relevant case of three–dimensional space, it describes the evolution of interfaces between solid phases of a system, driven by surface diffusion of atoms under the action of a chemical potential (see for instance [5]).
As one can expect, by its parabolic nature, there holds a short time existence and uniqueness result for the surface diffusion flow starting from a smooth hypersurface (possibly with self–intersections), proved by Escher, Mayer and Simonett in [4]. Furthermore, in the same paper, the authors also showed that if the initial hypersurface is $C^{2,\alpha}$–close enough to a ball with the same enclosed volume (hence embedded), then the flow exists for every time and smoothly converges to a translate of such ball. Later, this result was extended in [1] and [2] in dimensions two and three, to hypersurfaces close to boundaries of *strictly stable critical sets* for the volume–constrained Area functional, of which the flow is the gradient flow with respect to a suitable norm. In [3] we show that this result still holds in dimension greater than three, that is, for initial sets E sufficiently "close" to a smooth strictly stable critical set E, the flow exist for all positive times and asymptotically converges to a translate of E.
Moreover, there is evidence that this result can be possibly generalized also to other geometric flows which are "fractional" (nonlocal) gradient flows of the volume–constrained Area functional.

This is joint work with Serena Della Corte, Nicola Fusco and Carlo Mantegazza.

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Estimates for Robin p-Laplacian eigenvalues of convex sets with prescribed perimeter

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In this paper, we prove an upper bound for the first Robin eigenvalue of the p-Laplacian with a positive boundary parameter and a quantitative version of the reverse Faber-Krahn type inequality for the first Robin eigenvalue of the p-Laplacian with negative boundary parameter, among convex sets with prescribed perimeter.

The proofs are based on a comparison argument obtained by means of inner sets, introduced by Payne, Weimberger and Polya.

This is a joint work with Vincenzo Amato and Alba Lia Masiello.

Partially overdetermined problems in unbounded domains: first steps

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A generalization of Serrin's classical overdetermined problem consists in considering domains Ω inside some unbounded set $\mathcal{C}$ of $\mathbb{R}^N$ and prescribing both Dirichlet and Neumann conditions only on the part of $\partial\Omega$ inside $\mathcal{C}$. While the case when $\mathcal{C}$ is a cone has been widely studied, this poster presents some partial general results. A remarkable point is that a notion of relative (to $\mathcal{C}$) Cheeger set of Ω appears, so we also introduce this concept and show some related results. This is the theme of my ongoing PhD and joint work with Alessandro Iacopetti and Filomena Pacella.

References

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Linear Dynamics and Composition Operators

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In my poster, I present some topics I have been working on from Linear Dynamics, an area of mathematics consisting in the study of the behavior of the iterates of linear operators, lying in between Operator Theory and Dynamical Systems. I define some of the main notions, draw diagrams showing the implications occurring among them, and state some of the obtained results about chaotic and hyperbolic properties for a large and frequently recurring class of linear operators: the *composition operators,* the study of which consists in the relations between the dynamical behaviors of the linear operator $T_f : \varphi \to \varphi \circ f$ and the properties of the transformation f itself.

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Sharp and quantitative estimates for the $p-$Torsion of convex sets

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Let $\Omega \subset \mathbb{R}^n$, $n \geq 2$, be a bounded, open and convex set and let f be a positive and non-increasing function depending only on the distance from the boundary of Ω. We consider the $p-$torsional rigidity associated to Ω for the Poisson problem with Dirichlet boundary conditions, denoted by $T_{f,p}(\Omega)$.

Firstly, we prove a Pólya type lower bound for $T_{f,p}(\Omega)$ in any dimension; then, we consider the planar case and we provide two quantitative estimates in the case $f \equiv 1$.

This is a joint work with Amato Vincenzo, Paoli Gloria, Sannipoli Rossano.

Second Order Regularity for some Elliptic Systems with Discontinuous Coefficients

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We present regularity results for a linear elliptic system having the coefficients in BMO. As example we consider the elliptic system

$$div\, \Lambda\, [\, \exp(-|x|) - \log|x|\,]\, I\, Du = div\, F + g$$

in the unit ball. In this case our result reads that the gradient of the solution $u \in W_0^{1,2}$ to the corresponding Dirichlet problem is weakly differentiable provided the constant Λ is not large enough and F and g are sufficiently regular.

References

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Brittle Membranes in Finite Elasticity

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We derive a reduced model for brittle membranes in finite elasticity taking into account a non-interpenetration constraint. The main tools we develop for our analysis are: (i) a new density result in $GSBV^p$ of functions satisfying a maximal-rank constraint on the subgradients, which can be approximated by C^1-local immersions on regular subdomains of the cracked set, and (ii) the construction of recovery sequences by means of suitable $W^{1,\infty}$ diffeomorphisms mapping the regular subdomains onto the fractured configuration.

This is a joint work with S. Almi and F. Solombrino.

An isoperimetric inequality for the first Steklov-Dirichlet Laplacian eigenvalue of convex sets with a spherical hole

Joint paper with N. Gavitone, G. Paoli & G. Piscitelli

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In this paper we prove the existence of a maximum for the first Steklov-Dirichlet eigenvalue in the class of convex sets with a fixed spherical hole under volume constraint. More precisely, if $\Omega = \Omega_0 \setminus \overline{B}_{R_1}$, where B_{R_1} is the ball centered at the origin with radius $R_1 > 0$ and $\Omega_0 \subset \mathbb{R}^n$, $n \geqslant 2$, is an open bounded and convex set such that $B_{R_1} \Subset \Omega_0$, then the first Steklov-Dirichlet eigenvalue $\sigma_1(\Omega)$ has a maximum when R_1 and the measure of Ω are fixed. Moreover, if Ω_0 is contained in a suitable ball, we prove that the spherical shell is the maximum.

Unique continuation from a crack's tip under Neumann boundary conditions

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In this poster I will present the paper "Unique continuation from a crack's tip under Neumann boundary conditions" by Veronica Felli and G.S. upcoming on Elsevier, Nonlinear analysis.

We establish a strong unique continuation principle and derive local asymptotics of solutions to second order elliptic equations at the edge of a $(N-1)$-dimensional crack, with homogeneous Neumann boundary conditions prescribed on both sides of the crack. More precisely we consider the following elliptic problem with homogeneous Neumann boundary conditions on both sides of the crack Γ

$$\begin{cases} -\Delta u = fu, & \text{in } B_R \setminus \Gamma, \\ \dfrac{\partial^+ u}{\partial \nu^+} = \dfrac{\partial^- u}{\partial \nu^-} = 0, & \text{on } \Gamma, \end{cases} \tag{1}$$

where

$$B_R = \{x \in \mathbb{R}^N : |x| < R\} \subset \mathbb{R}^N, \quad N \geq 2,$$

Γ is a closed subset of $\mathbb{R}^{N-1} \times \{0\}$ with $C^{1,1}$-boundary, and the potential f satisfies suitable regularity assumptions. The boundary operators $\frac{\partial^+}{\partial \nu^+}$ and $\frac{\partial^-}{\partial \nu^-}$ in (1) are defined as

$$\frac{\partial^+ u}{\partial \nu^+} := -\frac{\partial}{\partial x_N}\left(u\big|_{B_R^+}\right) \quad \text{and} \quad \frac{\partial^- u}{\partial \nu^-} := \frac{\partial}{\partial x_N}\left(u\big|_{B_R^-}\right).$$

We study the Almgren frequency function $\mathcal{N}$ around the point 0 lying on the edge of the crack, defined as the ratio between the local energy function

$$E(r) := \frac{1}{r^N} \int_{B_r \setminus \Gamma} (|\nabla u|^2 - fu^2)\, dx$$

and the local mass or height

$$H(r) := \frac{1}{r^{N-1}} \int_{\partial B_r} u^2\, d\sigma,$$

i.e.

$$\mathcal{N}(r) := \frac{E(r)}{H(r)}.$$

The boundedness of the frequency function $\mathcal{N}$ will imply a strong unique continuation principle from the edge of Γ. Furthermore, the monotonicity properties of $\mathcal{N}$ will allow us to obtain energy estimates on weak solutions of (1). These estimates will be combined with a blow-up analysis to prove that any $u \in H^1(B_R \setminus \Gamma)$ weakly solving (1) behaves, asymptotically at the edge of the crack Γ, as a homogeneous function with half-integer degree of homogeneity.

New linking theorems and their applications to critical growth elliptic problems with jumping nonlinearities

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In this poster we will present some new existence results for critical growth elliptic problems involving jumping nonlinearities.
A preliminary part will be about the construction of the minimal and maximal curves of the Dancer-Fučík spectrum. Then, we will present some new abstract linking theorems and we will use them to obtain nontrivial solutions of our critical growth problems.
Since our abstract results are of independent interest and can be used to obtain nontrivial solutions of other types of problems with jumping nonlinearities as well, the last part of the poster will be devoted to the study of a particular critical growth problem in dimension $N = 2$.
These results are part of a joint work with Kanishka Perera (FIT).