Contemporary algebraic and geometric techniques
in coding theory and cryptography
Summer school — July 18-22, 2022
Università degli Studi della Campania "Luigi Vanvitelli"

Optimal locally recoverable codes with hierarchy from nested F-adic expansions

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Abstract

An important desirable attribute in a distributed storage system is the efficiency in carrying out repair of failed nodes. Among many others, two important metrics to characterize efficiency of node repair are *repair bandwidth*, i.e., the amount of data download in the case of a node failure and *repair degree*, i.e., the number of helper nodes accessed for node repair. While regenerating codes [2] aim to minimize the repair bandwidth, codes with locality [3] seek to minimize the repair degree. The focus of the present talk is on codes with locality.

Definition 0.1 *Let n be a positive integer, $C \subseteq \mathbb{F}_q^n$ be a linear code, and S be a subset of the set of indices $\{1,...,n\}$. We say that C can tolerate x erasures on S if, whenever there are x erasures on components of a codeword with indices belonging to S, the missing components can be recovered by looking at $|S|-x$ other coordinates in S.*

In this talk we construct new locally recoverable codes with hierarchy of locality sets.

Definition 0.2 *Let n,k,d,b,a,λ be positive integers with $n>k$. An $[n,k,d,b,a,\lambda]$ hierarchical locally recoverable code (HLRC) is an $[n,k,d]$-linear code such that*

- $(a+\lambda) \mid n$,
- $(b+1) \mid (a+\lambda)$,

- *the codeword indices are partitioned into ℓ distinct sets A_i, each of size $a+\lambda$, such that C tolerates λ erasures on A_i for every $i \in \{1,\ldots,\ell\}$, and*
- *each A_i can be partitioned into $B_{i,j}$, each of size $b+1$, such that C tolerates 1 erasure on each $B_{i,j}$ for every $i \in \{1,\ldots,\ell\}$ and every $j \in \{1,\ldots,(a+\lambda)/(b+1)\}$.*

Let us now briefly explain the motivation behind codes with hierarchical locality. Let T be the time needed to replace a failed node. Suppose that a second node fails in the same locality set of the first node during the time T. Now, an $[n,k,b,d]$ locally recoverable code will still need to access k information symbols, as the 1-locality cannot be used anymore. However, an $[n,k,d,b,a,\lambda]$-hierarchical locally recoverable code only requires accessing at most a information symbols. Since the scenario of failure of of a small number of $\lambda < d-1$ nodes is way more likely than the scenario of $d-1$ failures in the span of time T it is convenient to have a code that addresses separately the case of failure of two nodes. The codes in [1] address this issue, but they are restricted to a fixed λ, see [1, Theorem IV.2]. Morevoer, we explain how to construct such codes without relying on the arithmetic of power functions as in [5, Subsection 4C, Examples].

Our contribution In this talk we provide new constructions of optimal codes with hierarchical locality and an improved bound for H-LRC in our set of parameters. Our construction is based on the ideas in [1] combined with powerful techniques from algebraic number theory, that allow to prop arithmetic restrictions on the size of the hierarchy compared with q or $q-1$. Notice that our improved bound is very often equal to the one already present in the literature [4] but in the cases that interest us we gain a stricter bound on the distance (by just a -1 value) that will in turn guarantee that some more codes are optimal.

Keywords: Locally recoverable codes, Places, Optimal codes

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