Multinode Shepard method: from surface reconstruction to numerical solution of PDE's

D. Barrera[a], F. Dell'Accio[b], F. Di Tommaso[b], E. Francomano[c], F. Larosa[b], F. Nudo[b,d], J.F. Reinoso [e]

[a] Departamento de Matemática Aplicada, Universidad de Granada
[b] Dipartimento di Matematica e Informatica, Università della Calabria
[c] Dipartimento di Ingegneria, Università di Palermo
[d] Laboratoire de Mathématiques Appliquées de Pau, Université de Pau et des Pays de l'Adour
[e] Departamento de Expresión Gráfica Arquitectónica y en la Ingeniería, Universidad de Granada

dbarrera@ugr.es, francesco.dellaccio@unical.it, filomena.ditommaso@unical.it, elisa.francomano@unipa.it, f.larosa99@gmail.com, federico.nudo@unical.it, jreinoso@ugr.es

Let $\Omega \subset \mathbb{R}^d$, $d \geq 2$, be a region (i.e. a non-empty connected open set), $\partial\Omega$ its boundary, $\mathcal{X} = \{\boldsymbol{x}_i\}_{i=1}^n \subset \Omega \cup \partial\Omega$ a finite set of pairwise distinct nodes. Let $\mathcal{F} = \{F_j\}_{j=1}^m$ be a covering of $\mathcal{X}$ by non empty subsets $F_j \subset \mathcal{X}$. The multinode basis functions with respect to the covering $\mathcal{F}$ [1] are defined by

$$W_{\mu,j}(\boldsymbol{x}) = \frac{\prod\limits_{\boldsymbol{x}_i \in F_j} \|\boldsymbol{x} - \boldsymbol{x}_i\|^{-\mu}}{\sum\limits_{k=1}^{m} \prod\limits_{\boldsymbol{x}_i \in F_k} \|\boldsymbol{x} - \boldsymbol{x}_i\|^{-\mu}}, \quad j = 1, \dots, m, \quad \boldsymbol{x} \in \mathbb{R}^d,$$

where $\|\cdot\|$ denotes the Euclidean norm and $\mu > 0$ is a parameter that determines the differentiability class of functions $W_{\mu,j}(\boldsymbol{x})$. If, for each $j = 1, \dots, m$, we are able to provide a unique solution $P_j(\boldsymbol{x})$, to local interpolation problems on the nodes of F_j in some polynomial subspace of $\mathbb{R}[x_1, \dots, x_d]$, the multinode Shepard method [2] remains defined as follows

$$\mathcal{MS}_\mu[f](\boldsymbol{x}) = \sum_{j=1}^{m} W_{\mu,j}(\boldsymbol{x}) P_j(\boldsymbol{x}), \quad \boldsymbol{x} \in \mathbb{R}^d,$$

and interpolates at all points of $\mathcal{X}$. In this talk, we will discuss some interesting applications of multinode Shepard method to the problem of tridimensional reconstruction of geological surfaces from a large number of data coming from Digital Elevation Models [3] and to the numerical solution of Elliptic Problems of Partial Differential Equations on domains of various forms [4].

References

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