Patankar-type Linear Multistep Schemes

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Numerous real-world phenomena involve the interplay between processes of production and decay or consumption [1] and can be therefore modeled by positive and conservative Production-Destruction differential Systems (PDS). Patankar-type schemes are linearly implicit integrators specifically designed for PDS with the aim of retaining, with no restrictions on the stepsize, the positivity of the solution and the linear invariant of the system. In this work we extend the Patankar technique, already established for Runge-Kutta [2, 3] and deferred correction [4] methods, to multistep schemes. As a result, we introduce the class of Modified Patankar Linear Multistep (MPLM) methods, for which a thorough investigation of the convergence is carried out. Furthermore, we design an embedding procedure for the computation of the Patankar weights and prove the high order of convergence of the resulting MPLM scheme. A comparative study on the simulation of selected test cases highlights the enhanced performance of the MPLM methods with respect to other Patankar-type discretizations.

References

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