Problem-oriented numerical methods for PDEs models from applications

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ABSTRACT

We focus on the accurate and efficient numerical solution of models from applications consisting of Partial Differential Equations (PDEs), even in several spatial dimensions. Among them, we mention e.g. combustion models, models related to production of solar cells (Maldon et al. 2020), models of deterioration and corrosion of materials (Waschinsky et al. 2021), vegetation models (Eigentler et al. 2019). Usually, PDEs models from applications are characterized by a-priori known properties that it would be appropriate to preserve in the discrete setting, for large values of the space and time step-sizes. Furthermore, PDEs models can be characterized by high stiffness. It is therefore necessary to use non-trivial numerical techniques in order to efficiently compute the related solution.

In this talk, we show techniques for the construction of new problem-oriented numerical methods, which are efficient and stable, i.e. able to handle stiffness preserving the main properties of the solution (e.g. long term behavior, any positivity or oscillation frequency) for large discretization step-sizes. In particular, we show how the use of Time-Accurate and highly-Stable Explicit operators [1] leads to: a new class of stable parallelizable peer methods [2]; generalized NonStandard Finite Differences discretizations [3] for the adapted solution of a model for the growth of vegetation in arid environments [4]; a new class of efficient W-methods, especially advantageous for the solution PDEs in several spatial dimensions [5]. Numerical results testify the effectiveness of the proposed methods.

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