Symbol-based aggregated multigrid method for block Toeplitz linear systems

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We present novel improvements in the context of symbol-based multigrid procedures for solving large block structured linear systems. In detail we present a combination of an aggregation-based strategy that transforms the symbol from matrix-valued to scalar-valued with standard grid transfer operators for scalar Toeplitz systems. This allows for the study of sufficient convergence conditions for grid transfer operators and smoothing parameters for scalar-valued functions, which are more computationally tractable.

In addition, we complete the analysis with the study the block Jacobi smoother and we provide a rigorous proof of the smoothing property. This ensures the convergence and optimality of TGM method and we extend the convergence analysis to the V-cycle method.

The choice of the admissible smoother parameters which depends on the symbol and the first block of the matrix system brings computational advantages and simplifications in many different applications.

We test the proposed strategies both as standalone methods and as preconditioners for Krylov iterative methods. Among the numerical tests we consider the Toeplitz and Toeplitz-like linear systems stemming from the discretization with $\mathcal{Q}_p$ Lagrangian FEM approximation of a second order differential problem and B-spline discretization with non maximal regularity.