Stochastic geometric numerical integration: the backward error analysis perspective.

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Backward error analysis is a powerful tool to detect the long-term behavior of numerical discretizations, applied to conservative problems. In this work, we consider stochastic Hamiltonian systems, both in Ito and Stratonovich formulations. It is well-known that, in the former case, the expected Hamiltonian grows linearly in time, i.e., it satisfies the so-called trace law, while, in the latter, symplecticity of the exact flow is retained and, in case of one-dimensional noise, the conservation of the Hamiltonian appears along the exact trajectories. Then, we address our attention on providing estimates on the long-term trends of numerical Hamiltonians, derived from suitable time discretizations of such equations, highlighting their eventual ability of retaining the same kind of behavior visible along the exact solutions of the aforementioned differential problems. The investigation will rely on the construction of the stochastic modified equations, both in the weak sense (for the Ito case) and in the strong sense (in Stratonovich scenario). In particular, we show that stochastic symplectic schemes, applied to Stratonovich Hamiltonian system, are able to reproduce a Hamiltonian deviation that remains reasonable bounded in very large time windows. For non-symplectic discretizations, in the two scenarios, we discover that, in general, a growth of the expected Hamiltonian error is visible, destroying any hope of being structure-preserving. Finally, selected numerical examples will confirm all the theoretical findings. This is a joint work with Raffaele D'Ambrosio (University of L'Aquila).