PhD Courses 2023/24

PhD Program in Mathematics, Physics and Applications to Engineering

XXXIX cycle

Low-energy nuclear physics techniques and their applications

Lecturer: R. Buompane (UniCampania) (4 ECTS – 16 hours)

The methods developed to study fundamental nuclear physics have found a wide range of applications in fields as diverse as biology, archaeology, environmental analysis, medicine, materials analysis, etc. In addition, the development of accelerators has provided opportunities to optimise techniques and develop more sensitive methods.

The course aims to introduce the techniques and instrumentation of low energy nuclear physics and their application to applied physics, with particular emphasis on the production, transport and analysis of charged particle beams, charged particle dosimetry, Ion Beam Analysis (IBA) techniques and Neutron Activation Analysis (NAA).

Patent as an inventive research activity

Lecturer: A. Capece (UniCampania) (6 ECTS – 24 hours)

The course focuses on patenting principles, practices and strategies in the processes of intellectual property management and enhancement and technology transfer at national and international level.

The lessons aim to promote the exploitation of research results through the protection of Intellectual Property (IP), providing tools and methods on procedural forms and steps to structure patent applications.

Specifically, they will concern the principles of Intellectual Property protection, patent submission and evaluation procedures, information on how to retrieve data on existing patents (anteriority search) and the necessary bibliographical tools.

Syllabus

1. *Patent Making* - Theories – Laboratory

- The Patent between Invention and Innovation
- Forms and types of patents
- Definition and practice of the patent for industrial invention
- Patentability requirements

- Good practices for structuring a patent application for an industrial invention
- Procedure for depositing and/or extending the patent
- Patent searches and classification codes
- Definition and practice of the utility model
- Comparison between patent for industrial invention and utility model patent
- Registration of designs and models
- Rights on inventions
- Practical laboratory on Intellectual Property: use of databases and setting up patent proposals

2. *Processes, tools and best practices to structure patents and models* - Theories – Laboratory

- Impact of Intellectual Property on Research
- Overview of industrial invention patents examples
- Overview of designs and models examples
- Practical laboratory on intellectual property: simulation of patent research proposals
- Presentation and explanation of patent proposals
- Round table to discuss the proposed patent files

3. *Strategies and forms of patent valorisation* - Theories – Laboratory

- Tools and opportunities for Intellectual Property valorisation
- Presentation of Innovation Awards and Competitions: Start Cup Campania, National Award for Innovation
- Overview of examples and mock-ups of Business model and Business plan
- Spin off as a tool for research valorisation
- Overview of academic spin-off examples
- Facilitated finance instruments
- Practical laboratory on Intellectual Property: patent valorisation and technology transfer
- Structuring business models to simulate start cup and spin off creation

Spline models for regression analysis

Lecturer: R. Campagna (UniCampania) (4 ECTS - 16 hours)

Smoothing and interpolating spline models have attracted a great deal of attention in recent years and have been widely used in many areas of science and engineering, such as signal and image processing, computer graphics, and recently, geometric deep learning and neural networks. Particularly, Smoothing spline functions are a powerful tool in the functional analysis and regression framework, to model and predict data trends.

The course aims at introducing basic smoothing spline models, including polynomials and L-splines, and penalized splines, as well as an overview of more advanced models, including nonparametric nonlinear regression splines. Two models are described in detail: smoothing splines and regression splines. Penalized least squares regression models, and methods for regularization parameter selection are also discussed.

Some applications to real data are presented.

An interactive hands-on session where students will apply smoothing and regression splines to simple problems using Matlab is also included.

Syllabus

1. Motivating applications: Signal and image processing, Computer graphics, Geometric deep learning and neural networks
2. Spline functions: Parametric and nonparametric regression, Polynomial splines, Interpolating splines
3. Spline bases: Truncated power basis, B-splines
4. Smoothing and regression models: Smoothing splines, Regression splines, Penalized regression splines

Principle of non-Newtonian Fluid Mechanics (5 ECTS - 20 hours)

Lecturers: C. Carotenuto (UniCampania) - M. Minale (UniCampania)

The aim of the course is to teach the first concepts of the mechanics of non-Newtonian fluids and their characterization so to be able to choose the best constitutive equation for each material, homogeneous or heterogeneous. These skills are necessary for the analysis and design of conventional and innovative materials and related production processes.

Short introduction to rheology and its various fields of application.

Constitutive equations. Newtonian and non-Newtonian fluids. Classification of non-Newtonian behaviours: dilatant, pseudo-plastic and plastic.

Modelling: Principles of continuous mechanics. Pure viscous fluids: Reiner-Rivlin fluid, Generalized Newtonian fluid; Viscoelastic Fluids: Maxwell Fluid, Simple Fluid, Coleman and Noll Second Order Fluid, Fading Memory.

Rheometry: Classification of rheometers. Rotational and capillary rheometers. Equation of rheometers: Simple Shear, Small-gap Couette, Cone-Plate, Plate-Plate, Large-gap Couette, Capillary Viscometer.

Experimental techniques, time-temperature superposition. Dynamic-Mechanical Spectroscopy: Analysis of viscoelastic properties of materials with oscillatory experiments.

Heterogeneous materials: Overview of the constitutive properties of emulsions (cosmetic creams, mayonnaise, polymer blends, etc.), suspensions (slurries, peanuts butter, etc.), gel (gelatin) and foams (expanded polystyrene, shaving foam, cream, etc.).

Pyro-electrohydrodynamics and advanced technologies for soft-matter manipulation (4 ECTS - 16 hours)

Lecturers: S. Coppola (ISASI) - S. Grilli (ISASI)

The course provides the description and exploitation of innovative fabrication methods for the manipulation of liquids, polymers, and high viscous materials. In particular, the method based on the pyro-electrohydrodynamics (pyro-EHD) effect will be presented focusing on different materials, high resolution printing and patterning properties. The main feature of the method discussed stands in the non-contact and nozzle free modality that allows to manipulate starting drops of the material of interest in a direct way. The course will be focused on the theory of the pyroelectric effect and its experimental exploitation for the manipulation of soft matter, opening towards its feasible application in different technological fields. A cross-overview with the advanced conventional technologies will be provided while the main properties and advantages of the pyro-EHD will be discussed for biomedical application, additive manufacturing of 3D microstructures and for functionalization of microfluidic lab-on-chip devices.

Theory of nuclear forces and nuclear matter (4 ECTS - 16 hours)

Lecturer: L. Coraggio (UniCampania)

The goal of this course is to introduce PhD students to our present knowledge of the theory of nuclear forces. First, the basic phenomenological features of the nuclear potential are presented, and their connection to the main aspects of strong force. Then, we start to follow the path that from the Yukawa potential, through models based on the meson theory, historically leads to the present approach to the derivation of two- and three-body nuclear forces which are rooted in the QCD by way of the effective field theory. Last section is devoted to study the nuclear environment that is considered the best testing ground for models of nuclear forces, that is the infinite nuclear matter. To this end, basic knowledge of the derivation of the equation of state of nuclear matter in terms of the Brueckner theory will be provided to the students.

Numerical methods for smooth and non-smooth optimization (4 ECTS – 16 hours)

Lecturer: S. Crisci (UniCampania)

Numerical optimization is a very active field of research, encompassing diverse areas of mathematics and having strong impact on a wide range of applications, e.g., in physics, chemistry, engineering, decision science and data science, where many problems can be formulated as the minimization of functions, possibly subject to constraints. The aim of this course is to provide an overview on numerical methods for smooth/nonsmooth optimization in both constrained and unconstrained frameworks. Optimality conditions and fundamentals of selected first- and second-order algorithms will be presented, with the final goal of making the students able to select and efficiently apply these methods.

An Introduction to Lynear Dynamics (4 ECTS- 16 hours)

Lecturer: E. D’Aniello (UniCampania)

The course begins providing the students with fundamental concepts of (not necessarily linear) dynamical systems. It focuses on the Birkhoff transitivity theorem and a close study of various properties related to chaos. Then all the given notions and results are revisited in the linear context.

Introduction to Set Theory (4 ECTS – 16 hours)

Lecturer: P. D'Aquino (UniCampania)

We will introduce the axioms of Zermelo Fraenkel (ZF), and develop the theory of ordinals, cardinals and their arithmetic. The transfinte induction will be also presented. Constructions of models of ZF. Axiom of choice and its equivalents. Independence results in set theory, in particular we will show the independence of the axiom of choice and of the continuum from the axioms of ZF. The constructible universe due to Godel in 1936, and the forcing method introduced by Cohen in 1963 will be analyzed. Some consequences of the independence of the continuum hypothesis in topology, measure theory and algebra will be discussed. Cofinality and inaccessible cardinals.

Model theoretic analysis of algebraic structures (4 ECTS – 16 hours)

Lecturer: P. D’Aquino (UniCampania)

Model theory is a branch of mathematical logic. The course will be focused on the main model theoretic techniques and tools in model theory as compactness, ultraproducts, elimination of quantifiers, model completeness, saturation, elimination of imaginaries. Elimination of quantifiers implies a good understanding of the definable sets in a structure. We will analyze the above properties in some algebraic structures.

Physics for Space Application (4 ECTS - 16 hours)

Lecturer: M. De Cesare (CIRA)

The course provides the basis of the experimental methodologies concerning the problems of measurement applications, diagnostic and theoretical-experimental characterization in aerospace application, typical of the re-entry phase (terrestrial and planetary). The need to qualify and measure on large on-ground laboratories for the development of modern diagnostic aerospace technologies is underline.

Research in mathematics education (4 ECTS - 16 hours)

Lecturer: U. Dello Iacono (UniCampania)

The course aims at supplying PhD students with the main theoretical frameworks in mathematics education and the main methodologies, by setting they in the historical context and in the national and international researches and by dealing the conceptual questions by an epistemological point of view.

In addition, the course aims to stimulate a critical analysis of the main teaching methodologies, also referring to the specific role of the teacher, to the conceptual, epistemological, linguistic and didactic aspects for the mathematics teaching and learning.

Teaching methods: multimedia lessons, laboratory activities, discussion of scientific papers.

Optics and Photonics for advanced multimodal metrology (4 ECTS - 16 hours)

Lecturer: P. Ferraro (CNR-ISASI)

Optical and photonic methods (interferometry, spectroscopy, holography, 3D imaging, IR, etc.) have the inherent advantage of being non-invasive, full-field and often based on image output. The course will address the fields of modern metrology based on optical and photonic approaches and methods for the characterization of materials, processes and components in the new paradigm of Industry 4.0. It will be given the groundwork for understanding the basic operating principles of the most advanced technology currently available for inspection and testing. Particular emphasis will be given to the interpretation and analysis of the measurements. Examples of applications in different fields (automotive, aerospace, cultural heritage, biotech, etc.) will be illustrated and discussed to understand the significant role of these methods nowadays and in future, taking into account the emerging "multimodal" approach in metrology. Finally the importance of exploiting Deep Learning in metrology connected to the aforementioned tools will be illustrated and discussed.

Stability analysis of open-channel flows with Newtonian and non-Newtonian fluids (5 ECTS - 20 hours)

Lecturer: M. Iervolino (UniCampania)

The course concerns the basic concepts of hydrodynamic instabilities, with application to a class of one-dimensional free surface flows which are encountered in both environmental and industrial applications. The governing equations for the one-dimensional free surface flow of thin layers are preliminarily discussed, with special reference to the rheological behavior of the considered fluid, i.e. Newtonian or non-Newtonian. Subsequently, the standard method of normal-mode analysis is applied to the investigation of the stability of the equilibrium flow of a thin-layer of fluid. The main features of unstable-free surface perturbations are evaluated based on the results of the normal mode analysis and their implication in practical applications is discussed. The theory of near-front wave expansion is then introduced to analysis the stability of flow influenced by the boundary conditions or the non-linear growth of the fronts of unstable perturbations, in a rather general framework that allows the application of this method to an even wider class of flows. In the last part of the course, PhD students are guided to the application of these techniques to examples from their own research field.

New Concepts and Materials for Applications in Photovoltaics, Energy Storage and Electronics (5 ECTS - 20 hours)

Lecturer: G. Landi (ENEA)

The course introduces:

- **New concepts and materials** for the next generation of **photovoltaics**: multijunction solar cells, multiple excitation solar cells, intermediate band solar cells and related technologies (for quantum dots, thin films, organic and perovskite).

- An **overview** of the latest advancements in **different types of batteries** (including rechargeable lithium and lithium-ion batteries, metal-air batteries) and supercapacitors with a comprehensive review of materials and technologies. Particular attention to environmentally friendly energy storage devices is given.

- **Exploration of new biodegradable, polymeric and organic materials** that can be used as alternative systems to the inorganic materials for biodegradable/transient **electronics applications** (which can physically disappear after a period of stable operation with harmless end products).

Biophotonics for clinics and environment (4 ECTS - 16 hours)

Lecturers: M. Lepore (UniCampania) - I. Delfino (Università della Tuscia)

The course will deal with the application of non-invasive optical techniques to the development of new diagnostic strategies and environment monitoring tools. Vibrational, fluorescence and scattering-based optical spectroscopies will be presented for investigating biofluids, human tissues and cells exposed to physical and chemical external agents, and enzymes in order to monitor biological processes and to develop new sensing schemes and devices.

Stochastic Processes and Analysis of Correlations (4 ECTS - 16 hours)
Lecturers: E. Lippiello (UniCampania) – A. Sarracino
The purpose of these lectures is to give a simple mathematical introduction to the description of stochastic processes with innovative applications in the field of epidemiology and earthquake data time- series analysis.

Syllabus

Markov processes. Master and Fokker Plank equations. Stochastic energetics. Branching processes. Watson-Galtonmodel. Application to genetics. Epidemic models. Applications to epidemiology and earthquake occurrence. Analysis of correlations in stochastic signals. Detrended Fluctuation Analysis. Power spectrum of a signal.

Biophysical mechanisms and therapeutic implications of human exposure to ionising radiation (5 ECTS - 20 hours)

Lecturer: L. Manti (Università di Napoli Federico II)

Human exposure to ionizing radiation (IR), as a result of both naturally occurring sources as well as from diagnostic and therapeutic applications, is ubiquitous and entails well-known risks along evident benefits. The aim of the course is to provide the basic knowledge of the mechanisms that govern the biological action of IR, starting from the strong link between the patterns with which energy is deposited within the biological target and the consequences these might have at cellular, tissutal and organismal level. In fact, IR is unique among all the mutagenic and carcinogenic agents because it gives rise to a peculiar distribution of ionization clusters at the nanometer level, whose spatio-temporal proximity determines the severity of the damage incurred by the most important macrobiomolecule, the DNA. A cascade of complex pathway is then triggered that process such damage, driving the cell towards restoration of its genomic integrity or to death by several modes or, towards transmission of heritable damage. The latter is the most hazardous scenario for long-term effects such as cancer onset. The course will then illustrate the main biophysical models currently describing and quantifying the biological action of IR and the experimental work that has allowed to lay the foundations for modern radiotherapy (RT), such as the concepts of dose fractionation, together with novel phenomena that have questioned the central dogma of DNA as the sole target of radiation action. Special attention will be also devoted to illustrate the most advanced frontiers of novel radiation-based strategies to improve cancer control and minimize damage to the ineludibly exposed normal tissue, thereby reducing the risk of secondary cancers. Specifically, hadrontherapy (the use of accelerated particle beams), FLASH-RT, laser-driven particle acceleration, and radioimmunotherapy and the exploitation of nuclear physics reactions to locally enhance the effectiveness of external particle therapy will be discussed.

Navier-Stokes equations: an introduction to the well(ill)-posed initial boundary value problem (5 ECTS – 20 hours)

Lecturer: P. Maremonti (UniCampania)

The non-stationary Navier-Stokes equations are a model for the dynamics of a Newtonian incompressible viscous fluid. This model is considered suitable in some applicative fields. The model arises to avoid some mathematical paradoxes that we meet with the Euler model of ideal fluids. In two dimensional domains the mathematical theory can be considered sufficiently achieved.

One proves not only that the PDE-problem is well-posed, but one achieves results also for some special phenomena, for example: time periodic motions under the action of time periodic forces, steady motions with reasonable physical conditions. Instead, the mathematical theory for the tridimensional model is unsatisfactory, one cannot say if the PDE-problem is well-posed or not. Many efforts in the last fifty years are made in one sense and in other sense with no concrete result.

The aim of the course is two-fold. From one side, we give the mathematical results in the two-dimensional case, pointing out the difficulties that one meets trying to extend the results in 3D. In particular in 3D we prove how in the case of "small data" everything works and, with methods of the energy, how it is possible to formulate and to find results of stability and attractivity of some steady motions. From another side, we just state the attempts devoted in proving the ill-posedness of the model.

Petri Nets and their applications in science and engineering (5 ECTS - 20 hours)
Lecturer: S. Marrone (UniCampania)

Petri Nets is a formal language introduced in 1962 in the PhD thesis of Carl Adam Petri. Starting from this date, they proved their capability of modelling both discrete and continuous systems, being able to create a wide scientific literature, a meaningful set of industrial applications and the consequent releasing of a huge number of tools for their modelling and analysis.

Using Petri Nets, it is possible not only to obtain qualitative information on the modelled system as liveness, presence of deadlock and stability but also to get quantitative information as the probability of staying into a particular state of the system.

Up to now, Petri Nets are an assessed modelling formalism that can be used by the scientists to model the system under their study. Since their introduction, different variants and dialects of such a formalism have been introduced to raise the expressive power and to ease the modelling task.

Among such derived formalisms: the Generalized Stochastic Petri Nets (where activities can cost stochastically distributed times), the Fluid Stochastic Petri Nets (where resources can be continuous as well as discrete) and the Stochastic Well-formed Nets (adding “colours” to the tokens).

The objective of the course is twofold. On one hand, it introduces such this formalism since its mathematical foundations showing both the syntax and the semantics of the language as well as the main methods for the qualitative and quantitative analysis. On the other hand, it fills the gap between theory and practice of the application showing pragmatic application cases of the formalism in different aspects of science and engineering: from the security of computer-based systems to performance of industrial plants, to the modelling of continuous physical phenomena.

Isotope Physics and Methodologies (4 ECTS - 16 hours)

Lecturer: F. Marzaioli (UniCampania)

The course “Isotope Physics and Methodologies (IPM)” will be developed onto a 24 hours pathway.

During the course the most important issues regarding the isotope sciences will be covered. In details, among the others, the most important issues such as i) a general overview of the isotope nomenclature for both stable and radioactive nuclides; ii) the most important isotope fractionation mechanisms; iii) the approaches and methodologies utilized to address research issues will be covered with a special emphasis onto Accelerator based Mass Spectrometry and data reduction/analysis. Opportunities of Laboratory experience(s) will also be planned aiming to apply acquired knowledge.

Natural Language Processing: State-of-Art, Tools and Open Challenges (5 ECTS - 20 hours)

Lecturer: F. Marulli (UniCampania)

Natural Language Processing (NLP) has emerged as a transformative field within the broader realm of artificial intelligence, revolutionizing how computers interact with and process human language.

NLP's significance in current research is undeniable, permeating a diverse range of disciplines and fueling groundbreaking advancements. Its ability to comprehend, analyze, and generate human language has paved the way for remarkable applications across various sectors, including healthcare, education, finance, and customer service. In healthcare, NLP-powered tools are facilitating medical diagnosis, drug discovery, and personalized treatment plans. In education, NLP is enabling adaptive learning platforms, personalized tutoring, and natural language-based question answering systems. In finance, NLP is enhancing fraud detection, risk assessment, and customer service interactions. And in customer service, NLP-based chatbots are providing 24/7 support, analysing customer sentiment, and automating routine tasks. The ever-expanding applications of NLP in current research underscore its transformative potential, promising to shape the future of human-computer interaction and usher in a new era of innovation

This course aims to provide to students interested in NLP and human-machine-interaction which is the current state of the art in this field, by providing:

- Basic theory notions about the most recent methods and techniques for modelling a NLP-based system;
- Basic practical examples and exercises to learn how to use open-source tools to implement NLP-based systems;
- Overview of the next generation of NLP systems, including Generative AI -based systems.

The course will be composed of theory lectures and laboratory (practical) activities.

Label free phase contrast microscopy: principles and applications (4 ECTS - 16 hours)
Lecturers: L. Miccio, V. Bianco (CNR-ISASI)

Label-free microscopy techniques exploiting the quantitative phase-contrast paradigm will be presented. In particular the methods based on interferometry and digital holography will be treated extensively. Physical principles will be reviewed and great attention will be given on the more efficient experimental arrangements and the most upgraded image processing procedures. The main feature of interferometric methods stands in the quantitative evaluation of the phase shift introduced in the wavefront due to the presence of a sample. This is the reason for the great range of applications of such techniques from macroscopic length scale to microscopic world. The course will be mainly focused on microscopy and, in particular, on the recent developments in the field of biomedicine.

Astrophysics with ultra-high-energy neutrinos and Neutrino Telescope (2 ECTS - 8 hours)

Lecturer: P. Migliozzi (INFN)

Meson production, atmospheric neutrinos, the discovery of high-energy neutrinos, Sources of astrophysical neutrinos, Cosmic neutrino flux estimates, Neutrino detection principle and event topologies, The need for km3 neutrino telescopes, Water and ice properties, Operating neutrino telescopes, Results from neutrino telescopes.

Computational solid and structure mechanics: Finite elements and Boundary elements (5 ECTS - 20 hours)

Lecturer: V. Minutolo (UniCampania)

The noun structure designates the objects that in nature are responsible for bearing loads; every object, in a sense, is a structure even if some object has the structural ability as its main characteristic and other does not. For instance, bones of vertebrates, beams, and rods in machines and buildings are eminently structures. The Earth's surface, a mountain slope, the skin, and a blood vessel behave as structures when they are called to support loads even if their principal duty is somewhat else.

The course deals with the computational formulation of the mechanics of solids and structures.

After a brief introduction on the mechanics of structures within the framework of the continuum mechanics, the discretization techniques with finite elements are described. First, the one-dimensional problem is treated; furthermore, two and three-dimensional description f the structure is afforded.

The fundamentals of the variational approach and Galerkin formulation are addressed.

Betti reciprocal theorem constitutes the base of the Boundary Element formulation for linearly elastic structures. The feasibility of the method with respect to the two-dimensional and the three-dimensional structures is highlighted. The property of self-adjointness of the elastic equilibrium operator is described as the principal protagonist of the derivation of the Boundary Integral Equations of linear elasticity. Moreover, the extension of the formulation to non-linear elasticity, anisotropy, and plasticity is addressed.

Several examples using Matlab coding are discussed and implemented.

The application of linear elasticity, limit analysis of structures, fracture mechanics, and elastic instability form the core of the course.

Numerical Applications for Physics and Engineering (5 ECTS - 20 hours)

Lecturer: B. Morrone (UniCampania)

Physical phenomena can be described by using different mathematical models. "Model" is a set of equations and/or other mathematical relationships able to capture the patterns of the events and then describe, forecast and control them. General laws and constitutive relationships are the main pillars of the mathematical models. In industrial activities mathematical modelling has become largely widespread, followed by analysis and numerical simulation. Ordinary (ODEs) as well as partial differential equations (PDEs) result from the applications of models in the Engineering and Physics fields. The course gives a glimpse of the most employed numerical methods for solving either ODEs or PDEs, focusing also on their implementation. Massive use of Matlab® is accomplished to test the different methods and their programming methods during the course interactively and several examples using Matlab coding are discussed and implemented.

<u>Syllabus</u>

• Short introduction to floating-point numerical type, significant digits, round errors and Taylor series. Introduction to Matlab programming.

• Ordinary differential equations. Introduction and motivations. Explicit and implicit Euler's method, Runge-Kutta methods, predictor-corrector method. Truncation errors. Examples for physics and engineering applications. Initial Value Problems (IVP) vs. Boundary Value Problems (BVP).

• Partial Differential equations: classification, physical examples and their meaning. Well-posedness. Steady and transient problems. Parabolic and elliptic equations. Finite difference methods and Finite Volume methods to solve PDEs. Stability problems for numerical methods of parabolic equations. Accuracy of the numerical solutions. Examples for physics and engineering applications.

• Numerical methods for solving linear systems of equations using iterative methods (Jacobi, GaussSiedel, SOR, SSOR).

Introduction to homogenization of elliptic equations (4 ECTS - 16 HOURS)

Lecturer: Francois Murat (Laboratoire Jacques-Louis Lions, Sorbonne Universit e, Paris, France)

Program: see the last page.

Semilinear elliptic problems: A variational Approach (5 ECTS – 20 hours)

Lecturer: B. Pellacci (UniCampania)

Semilinear elliptic problems arise in the description of various models in geometry, physics, mechanics, engineering and, more recently, in life sciences. Variational methods, as a branch or an evolution of the Calculus of Variations, are nowadays classical tools in the field of nonlinear differential equations. The course begins with the introduction of minimization techniques, as it is well- known that the simplest way to obtain a critical point of a functional is to look for a global extremum, which in most of the cases is a global minimum. Then the study addresses the case of functionals that are unbounded from below where minimization is replaced by constrained minimization or by minimax procedures. Then the focus will be concentrated on the Mountain Pass Theorem and the Saddle Point Theorem, each discussed with applications to the specific problems that motivated them.

Syllabus

0. Introduction: examples, models and motivations.
1. Mathematical background.
2. Minimization Problems: coercive functionals.
2.1. Constrained Minimization: minimization on spheres and the Nehari manifold.
3. The minimax principle; deformation arguments.
3.1. The Mountain Pass and the Saddle point Theorems.
3.2 The lack of compactness.

M. Badiale E. Serra, **Semilinear Elliptic Equation for Beginners**. Springer.

Ambrosetti, A. Malchiodi, **Nonlinear Analysis and Semilinear Elliptic Problems**. Cambridge University Press.

P.H.Rabinowitz, **Minimax Methods in Critical Point Theory with Applications to Differential Equations**. CBMS Regional Conference Series in Mathematics, vol. 65 American Mathematical Society.

M.Struwe, **Variational Methods. Applications to Nonlinear Partial Differential Equations and Hamiltonian Systems**, Springer.

M. Willem, **Minimax Theorems**. Birkhäuser.

An introduction to Reaction-Diffusion Equations (5 ECTS – 20 hours).

Lecturer: B. Pellacci (UniCampania)

Reaction-Diffusion equations (RDEs) constitute a widely used tool to model phenomena arising in applied sciences such as physics, biology or sociology.

The description of the diffusion of individuals in an ecosystem, or of genes in a population, to mention just a few examples, naturally leads to partial-differential equations (PDEs), which may include reaction terms (transformation, source, internal interactions) as well as diffusion.The main goal of the course will be to settle down a mathematical background on the classical initial-boundary value problems; then give a glimpse to some contemporary research lines in this field.

0. Introduction: examples, models and motivations.
1. Mathematical background and study of the initial boundary value problem.
1.1 Basic stuff on functional analysis, Sobolev spaces and eigenvalues.
1.2. Initial value problem: well-posedness both in bounded and unbounded domains; maximum and comparison principles.
1.3. Stationary solutions: existence, multiplicity, qualitative behavior; stability issues, principal eigenvalues.
1.4. Long-time behavior: convergence to equilibria, survival vs extinction.
2. A glimpse on contemporary research topics.
2.1. Optimization and shape optimization problems: optimal design of a habitat; best dispersal strategy.
2.3. Different diffusions, fractional derivative in time, cooperative and competitive systems.

R.S. Cantrell, C. Cosner, **Spatial Ecology via Reaction-Diffusion Equations,** Editore: John Wiley & Sons, Ltd
J.D. Murray, **Mathematical Biology,** Editore: Springer
S. Salsa, **Partial Differential equations in action,** Editore: Springer
J. Smoller, **Shock Waves and Reaction-Diffusion Equations,** Editore: Springer

Combinatorics and its applications (4 ECTS - 16 hours)

Lecturer: O. Polverino - F. Zullo (UniCampania)

Combinatorics is a branch of Mathematics of increasing importance, owing to its links with Information Theory, Statistics and other areas of Mathematics, such as Algebra and Geometry. This course will be a gentle introduction to the classical combinatorics and the new trends in Galois geometry, then focusing on some new recent aspects and some applications to Coding Theory and to Cryptography.

The topics of the course regard:

- Linear sets (Projection of subgeometries, Geometric and Algebraic field of linearity)
- Blocking sets (linear and non-linear, Rédei type, nuclei of pointsets)
- Applications (Coding Theory and Cryptography)

Some aspects of Brace Theory (5 ECTS- 20 hours)

Lecturer: A. Russo (UniCampania)

In 2007 W. Rump found a surprising relationship between *radical rings* and the *Yang-Baxter equation* (YBE for short), an important equation in theoretical physics and in statistical mechanics that has become the key tool in many areas of research, such as quantum groups, Hopf algebras, knot theory and tensor categories. Let X be a non-empty set. Consider a bijective map $r: X^2 \to X^2$ and write $r(x,y)=(\sigma_x(y), \gamma_y(x))$, where σ_x and γ_y are suitable maps $X \to X$. The pair (X, r) is called a *set-theoretic* (or *combinatorial*) *solution* of the YBE if

$r_1 r_2 r_1 = r_2 r_1 r_2,$

where $r_1 = r \times id_X : X^3 \to X^3$ and $r_2 = id_X \times r : X^3 \to X^3$. Also, (X, r) is said to be *non-degenerate* and *involutive* if the maps σ_x and γ_y are bijective and r^2 is the identity map of X^2. In order to characterise completely involutive non-degenerate solutions of YBE, Rump introduced the notion of *brace*, an algebraic structure which generalizes that of radical ring. Recall that a *left brace* is a non-empty set B endowed with two group structures $(B, +)$ and $(B, \cdot)$ such that $(B,+)$ is an abelian group and $a(b + c) = ab - a + ac$, for every $a, b, c \in B$.

Afterward, L. Guarnieri and L. Vendramin (2017) have generalized left braces to *skew* left braces and these structures have been used to produce and study non-degenerate bijective solutions, *not necessarily* involutive of YBE. The aim of the course is to give an introduction to the theory of (skew) braces.

Statistical Methods in Experimental Sciences (5 ECTS - 20 hours)

Lecturer: F. Terrasi (UniCampania)

The course aims at providing the students with a deep understanding of the basic grounds of statistical methods used in the analysis of experimental data, allowing them to identify the most adequate to the problem under study and to correctly interpret the statistical meaning of the results of their application.

Syllabus

- The results of an experiment as samples of statistical populations
- Multidimensional statistical variables. Change of variables; correlation.
- Statistical estimators: bias; efficiency.
- Maximum likelihood estimators.
- Hypothesis testing
- Least squares. Linear and non linear fits.

Digital Signal Processing (5 ECTS - 20 hours)

Lecturers: L. Verde (Unicampania)

Signal processing is a well-assessed discipline whose objective is to provide unifying methods to analyse and manipulate analog and digital signals as they are produced/consumed by systems. By studying these methods, the students are able to apply them in different domains: from biomedical to astrophysics, enabling the application of advanced filtering and processing stages as AI-based stages.

The signal processing course as the objective to introduce Ph.D. students to the theory and to provide practical methods for the analysis and the manipulation of digital signals. The course is structured to pursue three objectives.

- Introduction to the concept of the digital signals.
- The definition of practical algorithms for the manipulation of signals.
- Concrete definitions of the MATLAB tool suite for implementing the proposed algorithms.

The course will focus, in particular, on the signals and Simulink toolboxes. Students will be called to use these toolboxes to implement simple case studied of digital signal processing workflows.

Introduction to homogenization of elliptic equations

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Composite materials are widely used in industry and everyday life.

A composite material is a material produced from two or more constituent materials which have significantly different mechanical or physical properties and which are "merged together" through a fine mixture to create at a macroscopical level a "new" material whose properties are intermediate between those of its constituent materials. Typical "artificial" composite materials include iron-reinforced concrete and masonry, wood composites such as plywood and compressed wood, and fiberglass-reinforced polymers. Bone and wood are "natural" composite materials.

When the mechanical (or thermal) behaviour of the constituent materials is modeled by elliptic partial differential equations or systems, the mathematical homogenization theory aims to provide a framework for describing the mechanical (or thermal) behaviour of the resulting composite material.

From a mathematical standpoint, let Ω be a bounded open set of $\mathbf{R^N}$, and let α and β be two real numbers with $0 < \alpha \leq \beta < +\infty$. Consider a sequence of matrices $A^\varepsilon(x)$ which are equi-coercive and equi-bounded, and more precisely which satisfy, for every ε, $A^\varepsilon(x)\xi\xi \geq \alpha|\xi|^2$, a.e. $x \in \Omega$, $\forall \xi \in \mathbf{R^N}$, and $||A^\varepsilon||_{L^\infty(\Omega)^{N\times N}} \leq \beta$. For every $f \in H^{-1}(\Omega)$, consider the unique solution u^ε of the problem

$$u^\varepsilon \in H_0^1(\Omega), \quad -\operatorname{div} A^\varepsilon D u^\varepsilon = f \;\text{ in } H^{-1}(\Omega).$$

It is easy to prove that u^ε is bounded in $H_0^1(\Omega)$, and so one would like to extract a subsequence and to to pass to the limit in the equation above.

The main result of homogenization theory is that one can extract a subsequence, say ε', and that there exists an $\alpha-$coercive matrix $A^0 \in L^\infty(\Omega)^{N\times N}$, such that for every $f \in H^{-1}(\Omega)$, the sequence $u^{\varepsilon'}$ converges weakly in $H_0^1(\Omega)$ to the unique solution of the problem

$$u^0 \in H_0^1(\Omega), \quad -\operatorname{div} A^0 D u^0 = f \;\text{ in } H^{-1}(\Omega).$$

When considering composite materials, the matrix A^0 characterizes the homogenized composite material obtained from its constituent materials.

Moreover one can prove a "corrector result": there exists a matrix $P^\varepsilon \in L^2(\Omega)^{N\times N}$ such that the difference $Du^{\varepsilon'} - P^{\varepsilon'} Du^0$ converges to zero in $L^2(\Omega)^N$ strongly (and not only weakly) when u^0 is sufficiently smooth. The expressions $P^{\varepsilon'} Du^0$ and $A^{\varepsilon'} P^{\varepsilon'} Du^0$ are therefore explicit excellent approximations of the highly oscillatory vector fields $Du^{\varepsilon'}$ and $A^{\varepsilon'} Du^{\varepsilon'}$.

The above two results are only the two main results which can be proved in this field.

The plan of the course will be as follows:

– Motivation
– The cases of dimension one and of layered materials
– The main theorem: from any sequence of equi-coercive and equi-bounded matrices, one can extract a subsequence which H-converges
– Compensated compactness
– The corrector result; applications
– The linearized elasticity system
– The monotone case
– The periodic case
– Perforated domains with periodic holes and Neumann boundary condition
– Perforated domains with periodic or small holes and Dirichlet boundary condition

All the proofs will be given in details.

The course will not require anything else than the standard knowledge of the basic objects used to solve elliptic boundary value problems using Lax-Milgram lemma, Sobolev spaces and weak convergence.