Workhop "Geometric Analysis and PDEs in Sorrento"

Abstract

Weighted symmetrization for unweighted Robin problems

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We consider the p-Poisson equation with Robin boundary conditions, where the Robin parameter is a function. By means of some weighted isoperimetric inequalities, we provide various sharp bounds for the solutions to the problems under consideration. We also derive some Faber-Krahn-type inequalities.

Optimal embeddings for fractional-order Orlicz-Sobolev spaces

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The optimal Orlicz target space is exhibited for embeddings of fractional-order Orlicz–Sobolev spaces in the Euclidean space. An improved embedding with an Orlicz–Lorentz target space, which is optimal in the broader class of all rearrangement-invariant spaces, is also established. Both spaces of order less than one and higher-order spaces are considered. Related Hardy-type inequalities are proposed as well. This is joint work with A.Alberico, L.Pick and L.Slavikova.

Anisotropic p-Laplacian Evolution of Fast Diffusion Type

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The main model which will be discussed is an anisotropic, possibly non-homogeneous version of the evolution p-Laplacian equation, when fast diffusion holds in all directions. I will expose some recent results: in particular, the existence of a self-similar fundamental solution to this equation is shown, while uniqueness is proven in a smaller range. Moreover, the asymptotic behavior of finite mass solutions in terms of the self-similar solution will be sketched. The results are based on a joint work with J. L. Vázquez and B. Volzone.

Symmetrization for linear and nonlinear fractional elliptic problems

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We describe symmetrization results in the form of mass concentration (*i.e.* integral) comparison for fractional elliptic equations of the type $(-\Delta)^s u = f$ (0 < s < 1) in a bounded domain Ω , equipped with homogeneous Dirichlet boundary conditions. We use a new direct method which recovers, in the limit $s \to 1$, the classical pointwise Talenti rearrangement inequality. Some possible applications of the method to nonlinear equations and to equations with lower order terms will be discussed.

A spectral problem for the Laplacian in joined thin films

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We consider a 3d multi-structure composed of two joined perpendicular thin films: a vertical one with small thickness h_n^a and a horizontal one with small thickness h_n^b . We study the asymptotic behavior, as h_n^a and h_n^b tend to zero, of an eigenvalue problem for the Laplacian defined on this multi-structure. We shall prove that the limit problem depends on the value $q = \lim_n \frac{h_n^b}{h_n^a}$. Precisely, we pinpoint three different limit regimes according to q belonging to $]0, +\infty[$, q equal to $+\infty$, or q equal to 0. We identify the limit problems and we also obtain H^1 -strong convergence results.

It is a joint work with Delfina Gómez and Maria-Eugenia Pérez-Martínez (Universidad de Cantabria, Santander, Spain).

On Stokes eigenvalue problems

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We discuss Stokes eigenvalue problems, mostly under Navier boundary conditions. We show that for some domains the least eigenvalue is zero, which leads to the failure of the related Poincaré inequality. The full spectrum is then analyzed in (2D) squares and (3D) cubes.

On the uniqueness of positive solutions of the Lane-Emden problem

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The question of the uniqueness for the positive solutions of the Lane-Emden equation arose since the famous symmetry result by Gidas, Ni, Nirenberg (1979), which indeed implies uniqueness under Dirichlet boundary conditions when the domain is a ball. Many counterexamples to the uniqueness in suitable domains were then displayed till, during the eighties, a conjecture on the uniqueness when the domain is convex was formulated. Only partial answers to this conjecture have been given so far. In this talk we describe recent results about the asymptotic characterization of positive solutions, the non-degeneracy and the computation of their Morse index, that we have obtained in dimension 2. We then derive that the uniqueness conjecture in convex domains is true in the planar case, when the exponent of the power nonlinearity is large enough. We also discuss some recent uniqueness results for the fractional case.

Eigenvalues of the curlcurl operator on perturbed electromagnetic cavities

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We consider the curlcurl operator subject to homogeneous boundary conditions of electric type on a bounded domain of the Euclidean space and we study the dependence of the corresponding eigenvalues and eigenfunctions upon perturbation of the domain. The domain can be thought as an electromagnetic cavity since our differential problem is closely related to stationary Maxwell's equations. We present a few stability results and we address a shape optimization problem. Based on joint works with Michele Zaccaron.

Steiner symmetrization for anisotropic quasilinear equations

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I will present comparison results for quasilinear equations whose prototype is $-\Delta_{p,x}u - u_{yy} = f$ with homogeneous Dirichlet boundary conditions by Steiner rearrangement in variable x. The approach used to prove such a result is based on a discretization in y, and on the proof of a comparison principle for the discrete version of the auxiliary problem $AU - U_{yy} \leq \int_0^s f$, where $AU = (n\omega_n^{1/n}s^{1/n'})^p(-U_{ss})^{p-1}$. These results for $-\Delta_{p,x}$ have been extended to general operators of the form $-\operatorname{div}(a(|\nabla_x u|)\nabla_x u))$ where a is non-decreasing and behaves like $|\cdot|^{p-2}$ at infinity.

Monotonicity of solutions to $-\Delta_p u = f(u)$ in \mathbb{R}^N_+

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We consider weak distributional solutions to the equation $-\Delta_p u = f(u)$ in \mathbb{R}^N_+ under zero Dirichlet boundary condition. For a general class of nonlinearities f, we prove that any positive solution is monotone increasing in the direction orthogonal to the boundary of the half-space.

A rigidity result for the Robin Torsion problem

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Let $\Omega \subset \mathbb{R}^2$ be an open, bounded and Lipschitz set. We consider the torsion problem for the Laplace operator associated to Ω with Robin boundary conditions. In this setting, we study the equality case in the Talenti-type comparison, proved in [1]. We prove that the equality is achieved only if Ω is a disk and the torsion function u is radial. Moreover, we state also some rigidity results in the n- dimensional case and for the Poisson problem associated to the p-Laplace operator with Robin boundary conditions. This is a joint work with Alba Lia Masiello.

References

 A. Alvino, C. Nitsch, and C. Trombetti. A Talenti comparison result for solutions to elliptic problems with Robin boundary conditions, to appear on Comm. Pure Appl. Math.

Sobolev Anisotropic inequalities with monomial weights

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I will present some anisotropic Sobolev inequalities in \mathbb{R}^n with a monomial weight in the general setting of rearrangement invariant spaces (*e.g.* L^p , Lorentz, Orlicz, etc...). The monomial weights are defined by

$$d\mu(x) := x^{A} dx = |x_{1}|^{A_{1}} \cdots |x_{n}|^{A_{n}} dx,$$

where $A = (A_1, A_2, \dots, A_n)$ is a vector in \mathbb{R}^n with $A_i \ge 0$ for $i = 1, \dots, n$.

The results are contained in a jointly paper with F. Feo and J. Martin.

Wulff shape symmetry of solutions to overdetermined problems for Finsler Monge-Ampère equations

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I will discuss an overdetermined problem for a Monge-Ampère type equation, modeled upon general anisotropic norms H in Rn. Similarly to what happens in the Euclidean case, when the solution is subject to both a homogeneous Dirichlet condition and a second boundary condition, designed on H, the domain (and the solution) must have a Wulff shape symmetry associated with H. The talk is based on a joint paper with Andrea Cianchi.

Singular semilinear and quasilinear elliptic equations in half spaces

Dino Sciunzi

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I will discuss some recent results regarding the classification of positive singular solutions to pure singular semilinear or quasilinear elliptic equations $-\Delta_p u = 1/u^{\gamma}$ in the half space. This problem also arises from the study of the Hopf boundary Lemma for popular

This problem also arises from the study of the Hopf boundary Lemma for nonregular solutions to more general singular equations in bounded domains.