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Exploiting low-rank structures in the numerical solution of PDEs

Abstract

When solving PDEs over tensorized 2D domains, the regularity in the solution often appears in form of an approximate low-rank structure in the solution vector, if properly reshaped in matrix form. This enables the use of low-rank methods such as Sylvester solvers (namely, Rational Krylov methods and/or ADI) which allow to treat separable differential operators. We consider the setting where this global smoothness is absent, but still locally exists almost everywhere. We show that the solution can still be efficiently stored by replacing low-rank matrices with appropriate hierarchical low-rank structures, and Sylvester solvers can be generalized to this setting. The structure can be determined with a black-box approximation scheme, that finds it adaptively. In addition, theoretical results that guarantee the structure preservation hold for these more general structure as well, and the computational complexities of the proposed method nicely interpolate between the low-rank and the completely unstructured case. We discuss how to effectively evolve the structure in time when approximating the solution of the PDE at different time steps, in the hypothesis of moving (but isolated) singularities.