Nonnegative moment coordinates on finite element geometries

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In this paper, we introduce new generalized barycentric coordinates (coined as moment coordinates) on convex and nonconvex quadrilaterals and convex hexahedra with planar faces. This work draws on recent advances in constructing interpolants to describe the motion of the Filippov sliding vector field in nonsmooth dynamical systems, in which nonnegative solutions of signed matrices based on (partial) distances are studied. For a finite element with n vertices (nodes) in $\mathbb{R}^2$, the constant and linear reproducing conditions are supplemented with additional linear moment equations to set up a linear system of equations of full rank n, whose solution results in the nonnegative shape functions. On a simple (convex or nonconvex) quadrilateral, moment coordinates using signed distances are identical to mean value coordinates. For signed weights that are based on the product of distances to edges that are incident to a vertex and their edge lengths, we recover Wachspress coordinates on a convex quadrilateral. Moment coordinates are also constructed on a convex hexahedra with planar faces. We present proofs in support of the construction and plots of the shape functions that affirm its properties.