Appell-type Quadrature Formulas

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Abstract

We derive some quadrature formulas by replacing the integrand function with an interpolating umbral polynomial.

Let L be a linear functional on $C^k[a,b]$ with $L(1) \neq 0$ and Q a δ-operator on the space P_n of polynomials of degree $\leq n$. The *umbral interpolant* for the function f related to L and Q is the unique polynomial $p_n[f]$ of degree $\leq n$ satisfying

$$L(Q^i[f]) = L(Q^i[p_n[f]]), \quad i = 0, 1, ..., n, \quad \forall f \in C^k[a,b] \text{ such that } Q^i[f] \in C^k[a,b].$$

If Q is the δ-operator of differentiation $Q = \frac{d}{dx} = D$, we obtain the Appell family of umbral interpolants and hence the *Appell quadrature formulas*

$$\int_a^b f(x)dx \approx \sum_{i=0}^{n} \frac{L\left(f^{(i)}\right)}{(i+1)!} \left[a_{i+1}^L(b) - a_{i+1}^L(a)\right], \qquad \forall n \in \mathbb{N},$$

where a_i^L is an umbral basis defined by the functional L.

Related to the interpolant polynomials there are the so-called *complementary* interpolant polynomials, from which we get other quadrature formulas. Better approximations are achieved by using composite formulas.

For suitable choices of the functional L we obtain special quadrature formulas. In particular we get the Appell-Bernoulli and the Appell-Euler formulas. Finally, we provide estimations of the remainder and some numerical examples.

References

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