Rough approximate subgroups

Frank Wagner
Université Lyon 1

Let k be an integer. A subset A of a group G is a k-*approximate subgroup* if there is a finite set E of size at most k such that $A^2 \subseteq EA$. They were first studied by Freiman (1973) in the abelian setting; following work of Hrushovski (2011), finite approximate subgroups were classified by Breuillard, Green and Tao (2012). The first step of the proof is Hrushovski's Lie Model Theorem: Up to commensurability, an pseudo-finite (or definably measurable) approximate group has a quotient which is a locally compact Lie group. The original proof uses a *Stabilizer Theorem*, assuming the existence of an S1-ideal of negligible sets; an alternative version due to Sanders, Massicot and myself uses more of the measure but has better definability properties. In his thesis under the supervision of Hrushovski, Arturo Rodriguez Fanlo (2022) generalizes the Lie Model Theorem to *rough approximate subgroups*, where $A^2 \subseteq EAT$ for some A-invariant type-definable subgroup T (for instance an infinitesimal neighbourhood of the identity in a metric group). While Arturo again passes through a near-group theorem, we have recently been able to adapt the alternative approach using outer measures.

Joint work with Arturo Rodriguez Fanlo.