Differentially Large Fields

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A field K is large (or ample) if it is existentially closed in its Laurent series field $K((t))$. Most (but not all) fields that have some tameness property are large. Model theoretically, tameness could mean decidability, or, possessing a feasible elimination theory, or, having stable like behavior. Algebraically, tameness could be understood as saying that every class of fields that serves as a locality, in the sense that local global principles hold with respect to this class, consists entirely of large fields. Examples of large fields are C, R, Q_p, pseudo finite fields, pseudo-classically closed fields and fields that possess a proper henselian valuation. Differentially large fields are supposed to play a similar role in the class of differential fields (in m commuting derivations) of characteristic 0. A differentially large field is a differential field of characteristic 0 that is existentially closed in $K((t))$, $t = (t_1, . . . , t_m)$, furnished with the natural derivations extending the derivations on K. Examples are differentially closed fields, closed ordered differential fields (in the sense of M. Singer) and pseudo differentially closed fields. In the talk I will illustrate tame properties of large fields that transfer to differentially large fields (in the first place: the class is axiomatizable) and then give examples and constructions of differentially large fields via iterated power series. A particular focus will be given to a differential algebraic tool, namely a twisting of the classical Taylor morphism associated to a (not necessarily differential) ring homomorphism f between differential rings; the twist will extend the differential part of f. I will explain how the twisting is used to work in differentially large fields and show how to obtain differentially large power series fields K furnished with a (naturally defined) differential logarithm, i.e. a group homomorphism $\log : (K^{\times}, \cdot) \to (K, +)$ satisfying $\delta(\log(f)) = \delta(f)/f$.

This is joint work with Omar León Sánchez.