Model Theory of Modules over Prüfer Domains via their value groups

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An integral domain is Prüfer if its localisation at each maximal ideal is a valuation domain. Many classically important rings are Prüfer domains. For instance, they include Dedekind domains and hence rings of integers of number fields; Bézout domains and hence the ring of complex entire functions and the ring of algebraic integers; the ring of integer valued polynomials with rational coefficients and the real holomorphy ring of a formally real field. Much of the model theory of modules over a Prüfer domain R is captured by its value group, that is, the group of finitely generated fractional ideals of R ordered by reverse inclusion. This group is a lattice ordered abelian group and all lattice ordered abelian groups occur as the value group of a Prüfer, or even Bézout, domain. In this talk I will present how various standard measures of complexity from model theory of modules, for instance the m-dimension and breadth of the lattice of pp-formulae and the existence of superdecomposable pure-injective modules, are visible in the value groups of Prüfer domains.