GRUPPO DI RICERCA

Mathematical Models and PDEs in Mathematical Physics

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1. DATI IDENTIFICATIVI DEL GRUPPO DI RICERCA

Categorie ERC	PE1_11 Theoretical aspects of partial differential equations PE1_12 Mathematical physics
Settore Scientifico Disciplinare	MATH-04/A
Parole Chiave (Keywords)	**Fluid Dynamics** • Navier-Stokes equations: existence, uniqueness, suitable weak solutions, regularity, energy equality, steady flows, boundary data; • Non-Newtonian power-law models: global regularity, high regularity, extinction properties; **Elasticity theory** • elastic solids; viscoelastic solids; stress relaxation; creep; **Diffusion phenomena** • singular p-Laplacian system; p(t,x)-Laplacian system; global regularity; high regularity; Burgers equations.

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2. COMPOSIZIONE E COORDINAMENTO

Responsabile Scientifico / Coordinatore:

- **Nome e Cognome:** Paolo Maremonti
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Componenti del Gruppo:

1. **Anna Abbatiello** – RTD-B orcid.org/0000-0001-5758-114X
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3. ATTIVITÀ SCIENTIFICA E NETWORK

Breve Descrizione delle linee di ricerca:

The Mathematical Physics group at the Department of Mathematics and Physics of University of Campania "Luigi Vanvitelli" has an extensive experience in the analytic theory of Newtonian and non-Newtonian fluids, in Elasticity theory, and Mathematical Models for Continuum Mechanics which has led to a wide literature published in international journals over the years.

Fluid Dynamics: Navier-Stokes equations

The Navier–Stokes equations model the dynamics of incompressible viscous fluids and are among the most important tools for describing fluid phenomena, with widespread applications.

Despite their relevance, the mathematical theory of the three-dimensional Navier–Stokes equations remains largely open and has attracted intense interest in the PDE community over the past decades, as highlighted by the Millennium Prize problem.

The central question concerns the validity of the Hadamard principles of well-posedness. For any initial data with finite kinetic energy, the existence of a global-in-time weak solution is known. However, weak solutions lack sufficient regularity: unlike the two-dimensional case, uniqueness and smoothness are still unresolved, raising concerns about their consistency with the laws of continuum mechanics underlying the model.

From a mathematical and physical perspective, understanding the regularity of weak solutions is therefore essential. The main goal is to establish suitable regularity criteria. Several results based on scaling-invariant quantities are available, typically requiring smallness assumptions. Among them, the Caffarelli–Kohn–Nirenberg criterion plays a fundamental role, providing partial regularity results in space–time regions consistent with parabolic scaling. Further developments build upon this approach.

Related research also addresses existence, uniqueness, and regularity of steady solutions for various boundary value problems, including adherence and slip conditions, as well as for the associated Stokes and Oseen systems, under different assumptions on boundary regularity and data.

Additional research directions include fluid–structure interaction problems.

Fluid Dynamics: compressible Navier-Stokes equations

In accordance with the basic principles of classical continuum mechanics, the state of a fluid is fully characterized at a macroscopic level by the density, the velocity and the temperature.

These quantities satisfy a system of partial differential equations expressing the conservation of mass, momentum and energy. Under suitable assumptions on the constitutive laws the outcome may be the full system of the Navier-Stokes equations of a viscous compressible and heat-conducting fluid. Questions related to existence, uniqueness and stability are far from being settled even in two dimensions. When the conduction of heat can be neglected the original system reduces to the Navier-Stokes equations of a compressible barotropic fluid where the unknowns are the density and the velocity. Similarly to the well known incompressible Navier-Stokes system, the existence of global in time smooth solutions for any physically admissible and large initial data remains an outstanding open problem. It is known that the problem is well posed locally in time in the framework of classical solutions. The system has been also studied in the framework of weak solutions. The research group intends to investigate regularity criteria and also to understand the crucial role of boundary conditions in the analysis of the underlined systems. Open systems, where energy can be exchanged with the outer world, are characterized by more complex boundary conditions than the academic no-slip one.

Fluid Dynamics: Non-Newtonian power-law models

Numerous experimental studies show that, for many incompressible fluids such as blood, viscosity may either decrease or increase with the shear rate within suitable regimes.

As a consequence, the classical Navier–Stokes model is no longer adequate. Such fluids are better described as power-law fluids, whose generalized viscosity depends on the shear rate through a power law. An increasing or decreasing viscosity corresponds, respectively, to shear-thickening fluids (such as batter) or shear-thinning fluids (such as blood, latex paint, and lubricants). Their wide range of industrial, medical, biological, and engineering applications explains the growing interest in their mathematical modeling and highlights the crucial role of rigorous analysis in these contexts.

Despite their practical relevance and significant experimental advances, the mathematical theory of these models remains incomplete and is essential for their validation. While the existence of weak solutions is well established, results on uniqueness and global regularity are still only partial.

The research group intends to further develop the mathematical analysis of these models, focusing on improvements in uniqueness and regularity of solutions. In particular, attention will be devoted to continuity properties of the velocity gradient and to higher regularity results, understood as enhanced integrability of second-order derivatives, for solutions of boundary value and initial–boundary value problems. Due to the nonlinear nature of the operator, such regularity properties are especially challenging and noteworthy.

Elasticity theory

Members of the research group are interested in the study of elastic and viscoelastic bodies.

Research on elastic solids focuses on the analysis of solution properties for the equations of motion, with particular attention to their extension to unbounded body configurations.

Equilibrium problems for elastic bodies are also investigated, with the aim of establishing existence and uniqueness results for boundary value problems under displacement, traction, and other types of boundary conditions, as well as studying the spatial asymptotic behavior of solutions to the equilibrium equations.

Viscoelastic solids are also examined through the development of nonlinear models describing stress relaxation and creep phenomena in bodies whose material properties are typical of biological tissues.

Diffusion phenomena

Members of the group are interested in the study of the p-Laplacian system, both in the elliptic and parabolic settings. Since the main nonlinear operator is closely related to that arising in power-law fluid models, our primary focus is on high-regularity properties of solutions, with the aim of extending these results to fluid dynamics. In particular, we investigate whether the introduction of an additional dynamic variable, namely the pressure, together with the incompressibility constraint, allows one to preserve the regularity properties established for the corresponding elliptic or parabolic systems.

We are also involved in the study of p(x)- and p(t,x)-Laplacian systems. These problems naturally fall within the framework of the regularity theory for minimizers of variational integrals with non-standard growth. Similar systems also govern the steady and unsteady motion of electrorheological fluids.

Furthermore, we consider the Burgers equation with a p-Laplacian as the principal operator. The classical Burgers equation, where the Laplacian plays the role of the principal operator, is widely used both as a model for various applications and as an approximation of the Navier–Stokes system.

The aim is to investigate whether, in the nonlinear and possibly singular setting, fundamental properties known for the classical Burgers equation, such as the maximum principle, can still be established.

Collaborazioni Nazionali ed Internazionali:

- **Nazionali:** Politecnico di Milano; Università di Pisa.
- **Internazionali:** Virginia Tech (USA); Charles University of Prague (Czech Republic); Czech Academy of Sciences (Czech Republic); Albert-Ludwigs-Universität (Freiburg, Germany); Technical University of Darmstadt (Germany).

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4. PROGETTI, BREVETTI E PUBBLICAZIONI

Principali Progetti di Ricerca e Brevetti:

VISCOMATH – Università della Campania “Luigi Vanvitelli” (D.R. 111/2024 e D.R. 797/2024) – P.I. A. Tartaglione

Principali Pubblicazioni Recenti:

1. Abbatiello A., Basaric D., Chaudhuri N., On a blow-up criterion for the Navier-Stokes-Fourier system under general equations of state, **Nonlinear Analysis Real World Applications**, 2025. DOI: 10.1016/j.nonrwa.2025.104328
2. Abbatiello A., Bulicek M., Kaplicky P., *On the exponential decay in time of solutions to a generalized Navier-Stokes-Fourier system*, **Journal of Differential Equations**, 2024. DOI: 10.1016/j.jde.2023.10.036
3. Crispo F., Di Feola A.P., Grisanti C.R., *Estimates of a possible gap related to the energy equality for a class of non-Newtonian fluids,* **Nonlinear Analysis Theory Methods Applications***,* 2026. DOI: 10.1016/j.na.2026.114080
4. Di Feola A.P., Pane V.,Weighted estimates for the Stokes semigroup in the half-space, **Journal of Mathematical Analysis and Applications**, 2026. DOI: 10.1016/j.jmaa.2026.130390
5. Del Sarto G., Hieber M., Palma F., Zöchling T., *Time-periodic solutions to an energy balance model coupled with an active fluid under arbitrarily large forces*, **Nonlinear Analysis Real World Applications**, 2026. DOI: 10.1016/j.nonrwa.2025.104558
6. Maremonti P., Palma F., *The Motion of a Rigid Body in a Viscous Fluid: Results for Strong Solutions, Uniqueness and Integrability Properties*, **Journal of Mathematical Fluid Mechanics**, 2025. DOI: 10.1007/s00021-025-00977-5
7. Maremonti P., Pane V., *The Navier–Stokes Cauchy Problem in a Class of Weighted Function Spaces*, **Journal of Mathematical Fluid Mechanics**, 2025. DOI: 10.1007/s00021-025-00923-5
8. Paolo Maremonti, Francesca Crispo, Carlo Romano Grisanti, *Navier-Stokes equations: a new estimate of a possible gap related to the energy equality of a suitable weak solution*, **Meccanica**, 2023. DOI: 10.1007/s11012-023-01642-9
9. Tartaglione A., *On Isolated Singularities for the Stationary Navier-Stokes System*, **Journal of Mathematical Fluid Mechanics**, 2024. DOI: 10.1007/s00021-024-00905-z
10. Dubik J., Tartaglione A., Wineman A., De Vita R., *A finite strain integral model for the creep behavior of vaginal tissue*, **International Journal of Non-Linear Mechanics**, 2024. DOI: 10.1016/j.ijnonlinmec.2024.104729