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Preserving local structure of evolutionary differential equations

Abstract

Local conservation laws are among the most important features of a Partial Differential Equation. They state that the amount of the conserved quantity (e.g., mass, momentum, energy, charge) at a point or within an arbitrary small volume can only change by the amount of the quantity which flows in or out of the volume in the considered time interval. Requiring that a numerical method preserves a conservation law, implies that the scheme must satisfy strong local constraints.

Although methods that preserve invariants perform generally better than standard methods [1], when the solution of the problem is highly oscillatory, they still require very small time steps in order to correctly reproduce the oscillations of the solution. In the first part of the talk, we will present a novel numerical strategy for developing schemes able to both preserve local conservation laws and to efficiently solve highly oscillatory problems, such as breather solutions of wave equations [2,3].

In the second part of the talk, we shall consider a time fractional diffusion equation having conservation laws and provide sufficient conditions that a numerical scheme must satisfy to preserve them. We shall present in particular a new spectral time integrator for the time integration of such a problem and show its conservative properties [4].

Numerical tests will be shown to validate the theoretical results and to make some comparisons.

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This is a joint work with A. Cardone and D. Conte.

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[4] A. Cardone, G. Frasca-Caccia. Numerical conservation laws of time fractional diffusion PDEs. *Fract. Calc. Appl. Anal.*, **25**, 1459-1483 (2022).