A Nyström method for solving the Bagley-Torvik equation

M. C. De Bonis, and D. Occorsio

University of Basilicata, Via dell'Ateneo Lucano 10, 85100 Potenza, ITALY
mariacarmela.debonis@unibas.it

The aim of this talk is to propose a Nyström-type method for the numerical computation of the solution of the initial value problem (IVP)

$$\begin{cases} f''(y) + \lambda(D^{\frac{3}{2}} f)(y) + \sigma f(y) = \mu h(y), \\ f(0) = 0, f'(0) = \gamma, \end{cases} \tag{1}$$

where $\lambda, \sigma, \mu, \gamma \in \mathbb{R}$, h is a given function and

$$(D^\alpha f)(y) = \frac{1}{\Gamma(m-\alpha)} \int_0^y (y-x)^{m-\alpha-1} f^{(m)}(x) dx$$

is the Caputo's fractional derivative of order α with $m-1 < \alpha < m$ and $m = 2$.

The above IVP occurs quite frequently in various branches of applied mathematics and mechanics. It is called Bagley-Torvik equation because it was originally formulated by Bagley and Torvik in [1] for modelling the motion of a rigid plate in a Newtonian fluid. It was thoroughly discussed by Podlubny in [2] and in several papers (see, for example [2, 3, 4, 5]) techniques for the computation of its solution have been introduced.

We first consider the reformulation of (1) into an equivalent Volterra integral equation (VIE) of the second kind and, then, we propose a Nyström method for the global approximation of the solution of the VIE. Such method is based on the discretization of the integral operators defining the VIE by suitable product quadrature rules obtained using Lagrange interpolation.

Numerical tests showing the performances of the method will be presented, as well as comparisons with other numerical methods.

References

[1] R.L. Bagley, P.J. Torvik, On the appearance of the fractional derivative in the behavior of real materials, ASME J. Appl. Mech. 51 (1984) 294-298.

[2] Podlubny I. Fractional differential equations. San Diego: Academic Press, 1999

[3] K. Diethelm, N.J. Ford, Numerical Solution of the Bagley-Torvik equations, BIT, 2002, Vol. 42, No. 3, pp. 490-507

[4] K. Diethelm, The analysis of Fractional Differential equations, Springer Heidelberg Dordrecht London New York, 2010

[5] S. Saha Ray, R.K. Bera, Analytical solution of the Bagley-Torvik equation by Adomian decomposition method, Appl. Math. Comput. 168 (2005) 398-410.