A DATA-DEPENDENT REGULARIZATION METHOD BASED ON THE GRAPH LAPLACIAN

DAVIDE BIANCHI*, DAVIDE EVANGELISTA†, STEFANO ALEOTTI‡, MARCO DONATELLI‡, ELENA LOLI PICCOLOMINI†, AND WENBIN LI§

Abstract. We investigate a variational method for ill-posed problems, named `graphLa+`Ψ, which embeds a graph Laplacian operator in the regularization term. The novelty of this method lies in constructing the graph Laplacian based on a preliminary approximation of the solution, which is obtained using any existing reconstruction method Ψ from the literature. As a result, the regularization term is both dependent on and adaptive to the observed data and noise. We demonstrate that `graphLa+`Ψ is a regularization method and rigorously establish both its convergence and stability properties.

We present selected numerical experiments in 2D computerized tomography, wherein we integrate the `graphLa+`Ψ method with various reconstruction techniques Ψ, including Filter Back Projection (`graphLa+FBP`), standard Tikhonov (`graphLa+Tik`), Total Variation (`graphLa+TV`), and a trained deep neural network (`graphLa+Net`). The `graphLa+`Ψ approach significantly enhances the quality of the approximated solutions for each method Ψ. Notably, `graphLa+Net` is outperforming, offering a robust and stable application of deep neural networks in solving inverse problems.

*School of Mathematics (Zhuhai), Sun Yat-sen University, Zhuhai, 519082, China (bianchid@mail.sysu.edu.cn).

†Department of Computer Science and Engineering, University of Bologna, Bologna, 40126, Italy (davide.evangelista5@unibo.it, elena.loli@unibo.it).

‡Department of Science and High Technology, University of Insubria, Como, 22100, Italy (saleotti@uninsubria.it, marco.donatelli@uninsubria.it).

§School of Science, Harbin Institute of Technology, Shenzhen, Shenzhen, 518055, China (liwenbin@hit.edu.cn).

Funding: Davide Bianchi is supported by NSFC (Grant No. 12250410253). Marco Donatelli is partially supported by MIUR - PRIN 2022 N.2022ANC8HL and GNCS-INDAM. Wenbin Li is supported by Natural Science Foundation of Shenzhen (Grant No. JCYJ20190806144005645) and NSFC (Grant No. 41804096).