# Adaptivity beyond Coercivity

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## Abstract

The convergence and optimality analysis of adaptive finite element methods (AFEM) for variational problems of the form

$$u \in V \quad : \quad \mathcal{B}[u,v] = \mathcal{F}[v] \qquad \forall v \in V$$

is well established after the seminal paper by Cascón, Kreuzer, Nochetto, and Siebert (2008), when the form $\mathcal{B} : V \times V \to \mathbb{R}$ is symmetric and $V$-coercive. Beyond this case, very little is known.

A recent paper by R. Feischl allows us to handle the case of a variational problem of the form

$$u \in V \quad : \quad \mathcal{B}[u,w] = \mathcal{F}[w] \qquad \forall w \in W,$$

where $\mathcal{B} : V \times W \to \mathbb{R}$ satisfies an inf-sup condition

$$\exists \beta > 0 \quad : \quad \inf_{v \in V} \sup_{w \in W} \frac{\mathcal{B}[v,w]}{\|v\|_V \|w\|_W} \geq \beta.$$

The classical adaptive loop

$$\text{SOLVE} \longrightarrow \text{ESTIMATE} \longrightarrow \text{MARK} \longrightarrow \text{REFINE}$$

generates a sequence of meshes $\mathcal{T}$ and approximate solutions $u_\mathcal{T} \in V_\mathcal{T}$, which linearly converge to $u$, provided the sequence of subspaces $(V_\mathcal{T}, W_\mathcal{T})$ produced in the loop satisfies the same inf-sup condition above, uniformly in $\mathcal{T}$, and some assumptions on the error estimator are valid.

Building on this, we consider a two-loop adaptive algorithm which alternates the approximation of the data $\mathcal{D}$ of the problem and the approximation of the solution $u$. We prove that the algorithm has the finite termination property, and is quasi-optimal. This means that the error estimate

$$\|u - u_\mathcal{T}\|_V \leq C\,(\#\mathcal{T})^{-s}$$

holds true whenever the smallest approximation error of $u$ on meshes with $N$ elements decays as $N^{-s}$.

Applications to diffusion problems in mixed form and to the Stokes problem will be given. Limitations and open problems will be discussed as well.

This is a joint work with Ricardo H. Nochetto (University of Maryland).