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Book of Abstracts

Università
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Plenary Talks

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Title of the talk

Common hypercyclic vectors and dimension of the parameter set

Co-authors

Fernando Costa, Quentin Menet

Abstract

Let X be a Banach space. Let us recall that a bounded operator $T \in \mathcal{L}(X)$ is called hypercyclic if there exists a vector $x \in X$ with dense orbit and let us denote by $HC(T)$ the set of hypercyclic vectors for T. It is well-known that provided $HC(T)$ is not empty, it is a dense G_δ-set. Therefore, if Λ is countable, and if $(T_\lambda)_{\lambda\in\Lambda}$ is a family of hypercyclic operators on X, then $\bigcap_{\lambda\in\Lambda} HC(T_\lambda)$ is never empty.

One can ask what happens for (natural) uncountable families. The most classical examples is the family of multiples of the backward shift B acting on ℓ^2. It was shown by Abakumov and Gordon that $\bigcap_{\lambda>1} HC(\lambda B)$ is not empty whereas Borichev has observed that $\bigcap_{\lambda>1,\mu>1} HC(\lambda B \oplus \mu B) = \varnothing$.

Our aim in this talk is twofold.

1. we plan to understand precisely why the existence of a common hypercyclic vector breaks for the above two-dimensional family.

2. we want to explain how it is still possible to get common hypercyclic vectors for families indexed by a set with high dimension.

This will lead us to introduce a new notion of dimension of subsets of $\mathbb{R}^d$: the homogeneous upper box dimension.

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Title of the talk

Singularities of solutions to Hamilton-Jacobi equations: a long path from PDEs to topology, passing through geometric measure theory

Abstract

The study of the structural properties of the set of points at which a solution u of a first order Hamilton-Jacobi equation fails to be differentiable—in short, the singular set, or singularities, of u—has been the subject of a long-term project that started in the late sixties with a seminal paper by W. H. Fleming [5]. Research on such a topic picked up again after the introduction of viscosity solutions by M.Crandall and P.-L. Lions [1] (see also [2]) in the eighties and is still ongoing. All these years have registered enormous progress in the comprehension of the size and structure of singularities: a fine measure theoretical analysis of the singular set was developed, the dynamics governing propagation of singularities was identified, and connections with weak KAM theory by A. Fathi [3, 4] were pointed out. This effort also led to interesting topological applications. In this talk, I will revisit some of the milestones of the theory and describe its recent achievements.

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Title of the talk

Dynamics of zero dimensional systems and shifts of finite type

Abstract

Following a novel result of Good and Meddaugh, we discuss how a variety of dynamical properties of zero dimensional systems can be captured by shifts of finite type. The properties of interest, among others, are the shadowing property, transitivity and mixing. The dynamical systems themselves are $\mathbb{Z}^+$ flows as well as actions of finitely generated countable groups. We will also consider an instance when the phase space is not locally compact.

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Title of the talk

Ball-Evans approximation problem: recent progress and open problems

Abstract

In this talk we give a short overview about the Ball-Evans approximation problem, i.e. about the approximation of Sobolev homeomorphism by a sequence of diffeomorphisms (or piecewise affine homeomorphisms) and we recall the motivation for this problem. We show some recent planar results and counterexamples in higher dimension and we give a number of open problems connected to this problem and related fields.

We concentrate in detail on the joint result with A. Pratelli [1] about the approximation on planar $W^{1,1}$ homeomorphism by a sequence of piecewise affine homeomorphisms.

References

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Title of the talk

Extreme non-differentiability of typical Lipschitz mappings

Co-author

Michael Dymond

Abstract

The classical Rademacher Theorem guarantees that every set of positive measure in a finite-dimensional space contains points of differentiability of every Lipschitz function. A major direction in geometric measure theory research of the last two decades is to explore to what extent this is true for Lebesgue null sets.

In recent joint papers with Dymond we investigate this question from the point of view of differentiability of typical Lipschitz mappings. Here, 'typical' is understood in terms of Baire category.

Earlier, we showed that in a set that can be covered by countably many closed purely unrectifiable sets, a typical 1-Lipschitz real-valued function is nowhere differentiable, even directionally. In any other null set, a typical 1-Lipschitz function has many points of differentiability. Our most recent work shows, however, that in any set a typical point (and in 'coverable' sets as above every point), is a point of non-differentiability of a typical Lipschitz mapping, scalar or vector-valued, in a strong sense: the derivative ratios at the given point approach every operator of norm at most 1. The result about typical points holds for mappings between Banach spaces with arbitrary norms, while the finite-dimensional result about every point of a 'coverable' set is currently proved for a large class of norms but not all, which has led to an interesting problem in combinatorial geometry.

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Contributing Talks

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Title of the talk

On Baire One Path Systems and Differentiation

Abstract

Path differentiation was introduced by Bruckner, O'Malley and Thomson in [7]. A path leading to x is a set $E_x \subseteq \mathbb{R}$ containing x and having x as an accumulation point. A path system is a collection $E = \{E_x : x \in \mathbb{R}\}$ such that each E_x is a path leading to x. We say that F is path differentiable to f if there is a path system E such that for each $x \in \mathbb{R}$, $f(x) = \lim_{\substack{y \to x \\ y \in E_x}} \frac{F(y)-F(x)}{y-x}$, and is denoted by $F'_E(x) = f(x)$. The extreme path derivatives $\overline{F'}_E$ and $\underline{F'}_E$ are defined in the usual way. We introduced the concept of a continuous system of paths [1] and studied the extreme path derivatives as well as the path derived number of functions (see [1, 2, 3, 4]) in this setting. This concept was generalized as multifunctions by Milan Matejdes (see [11, 12, 13]).

Motivated by the poincare first return map of differentiable dynamics, R. J. O'Malley (see [15]) introduced the first return systems. He shows that, though these are extremely thin paths, the systems possess the intersection property. First return systems have been extensively investigated in a series of papers by U.B. Darji, M. J. Evans, P.D. Humke and R. J. O'Malley (see[8, 9, 10]) and some of their references. The first return path systems are not continuous. Using the idea of composite differentiation (see [16]) we introduced the notion of composite continuous path system (see [5]) as a generalization of continuous path system and studied the extreme path derivatives of continuous functions in this setting. Here we define the Baire one and Baire* one path systems and show that a path system E is composite continuous if and only if E is Baire* one. Here are some results.

Theorem 1. Every composite continuous system of paths is Baire one.

Theorem 2. Every first return path system is Baire 1.

Theorem 3. Let $E = \{E_x : x \in [0,1]\}$ be a Baire* one system of paths.
(i) If $f : [0,1] \to \mathbb{R}$ is a continuous function, then $\underline{f'}_E \in B_2$, $\overline{f'}_E \in B_2$.
(ii) If $f : [0,1] \to \mathbb{R}$ is a Borel (resp.; Lebesgue) measurable function, then $\underline{f'}_E$ and $\overline{f'}_E$ are also Borel (resp; Lebesgue) measurable functions.

Question. What can be said about the Baire classification of path derivatives and extreme path derivatives of continuous functions, Borel measurable functions or Lebesgue measurable functions when the path system E is Baire one?

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Title of the talk

Characterization of multiplier sets in BV-type spaces

Abstract

In this talk we will focus on multiplier sets, that is, the sets of all real-valued functions g, defined on a compact interval, such that the product fg belongs to some given function space for all f belonging to other (or the same) function space. It appears that in the case of some function spaces, finding their multiplier sets is easy whereas in other function spaces the problem is quite complicated. We will consider multiplier sets mainly in spaces of functions of bounded variation of various types. We will give the comprehensive answers to the problems concerning description of multiplier sets for the spaces of functions of bounded variation in the sense of Jordan, Riesz, Waterman, Wiener and Young.

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Title of the talk

Compactness in groups of group-valued mappings

Co-authors

A. Trombetta, G. Trombetta

Abstract

Aim of this talk is to obtain quantitative versions of theorems about compactness in groups of group-valued mappings, endowed with a topology which generalizes the topology of convergence in measure. Quantitative characteristics modeled on the concepts of extended equimeasurability and of extended uniform quasiboundedness allow us to estimate the Hausdorff measure of noncompactness in such a setting. We derive compactness criteria of Fréchet-Šmulian and Ascoli-Arzelà type.

References

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Title of the talk

On useful spaces in the study of quadratic fractional equations

Abstract

One of the classical issues in the theory of differential and integral equations is the regularity of solutions, i.e. in which function space solutions can be found. This, of course, depends on the problem, and here we will rearrange the problem for so-called quadratic problems, i.e., when there is a point product of operators in the equation. When we add to this the fact that these operators can be of fractional order, the problem gets complicated. We will propose a class of function spaces (more precisely: Banach algebras) as a natural solution to this problem.

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Self-similar fractals and common hypercyclicity

In collaboration with Stéphane Charpentier.

Abstract

In the core of common hypercyclicity theorems is the construction of a partition of a set of parameters. In 2004, G. Costakis and M. Sambarino obtained the first general result in this context partitioning a line segment by pieces of decreasing size. When it comes to sets of parameter in higher dimensions, the problems becomes very tricky. In this talk, we present a new way of discretizing a self-similar set of parameters with optimal consequences, which allows us to obtain a very general version of the Costakis-Sambarino criterion. Applications include many self-similar fractals and any Hölder curve.

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Title of the talk

A new decomposition for measures dominated by the Hausdorff measure $\mathcal{H}^s$

Co-author

Augusto C. Ponce (UCLouvain – IRMP)

Abstract

A theorem due to R. Delaware asserts that every Borel set E of finite s-dimensional Hausdorff measure may be decomposed as a countable union of Borel sets whose Hausdorff measure and content coincide. We generalize Delaware's result to general Borel measures μ satisfying $\mu \leq \mathcal{H}^s$ by decomposing $\mathbb{R}^N$ as a countable union of Borel sets on which the restriction of μ is bounded from above by the Hausdorff content. Such a decomposition has been applied to prove existence of solutions of a nonlinear PDE. This is joint work with A. Ponce (UCLouvain).

References

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Title of the talk

Categorical approach to graph limits

Co-author

Wiesław Kubiś

Abstract

The use of category theory in graph theory is quite common. We show that category theory may be useful even in the world of graph limits. To do so, we introduce a new category whose objects are certain generalizations of graphs where both distributions of vertices and edges are represented by abstract measures. This is a similar (but more general) approach as that of s-graphons introduced in [1]. A morphism in our category can be viewed as a 'fuzzy' map between the underlying spaces. The values of this map are not defined deterministically, we only know the probability that a given point is mapped to a given set. Formally, this idea is realized with the use of Markov kernels which, in a certain sense, preserve the distributions of vertices and edges. Further, we introduce a natural notion of convergence of sequences of graphs (or, more generally, of objects of our category) which is heavily inspired by s-convergence. Then we apply the categorical structure to show that each convergent sequence has a limit object.

References

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Title of the talk

Nemytzkij operators on function spaces of power weights and applications

Abstract

Let $G : \mathbb{R} \to \mathbb{R}$ be a continuous function. The corresponding composition operator T_G is defined by $T_G : f \longmapsto T_G(f) = G(f)$. In the first part of this talk, we present a necessary and sufficient conditions on G such that

$$\{G(f) : f \in W_p^m(\mathbb{R}^n, |\cdot|^\alpha)\} \subset W_p^m(\mathbb{R}^n, |\cdot|^\alpha)$$

holds, with some suitable assumptions on m, α and p. Under some assumptions on G, s, α, p and q we have that

$$\{G(f) : f \in A_{p,q}^s(\mathbb{R}^n, |\cdot|^\alpha)\} \subset A_{p,q}^s(\mathbb{R}^n, |\cdot|^\alpha) \tag{1}$$

implies that G is a linear function. Here $A_{p,q}^s(\mathbb{R}^n, |\cdot|^\alpha)$ stands either for the Besov space $B_{p,q}^s(\mathbb{R}^n, |\cdot|^\alpha)$ or for the Triebel-Lizorkin space $F_{p,q}^s(\mathbb{R}^n, |\cdot|^\alpha)$. These spaces unify and generalize many classical function spaces such as Sobolev spaces of power weights.

In the second part of this talk, we present a sufficient conditions on G such that (1) holds. An application, we will study local and global Cauchy problems for the semi-linear parabolic equations

$$\partial_t u - \Delta u = G(u)$$

with initial data in Triebel-Lizorkin spaces of power weights. Previous results in case $\alpha = 0$ are based on the translation invariance of the norms of the Sobolev, Besov and Triebel-Lizorkin spaces. The norm of the related weighted spaces with $\alpha \neq 0$ is not translation invariant. For this reason we are forced to introduce a new methods.

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Title of the talk

Riemann Integration and Asymptotic Structure of Banach Spaces

Co-author

Bunyamin Sari

Abstract

Let X be a Banach space. If every Riemann-integrable function $f : [0, 1] \to X$ is Lebesgue almost everywhere continuous, then X is said to have the Lebesgue property. A longstanding open problem in the geometry of Banach spaces is to derive a condition that is both necessary and sufficient in order for X to have the Lebesgue property. In this talk, I will give a brief overview of the history of work done on this problem and past results, and I will then present its recent solution (due to B. Sari and myself, and independently to M. Pizzotti). It turns out that the Lebesgue property is equivalent to an asymptotic structure in X that is strictly between the classical notions of spreading and asymptotic models. I will also discuss several other results (some due to B. Sari and myself) that help to place the Lebesgue property in its proper context with respect to asymptotic structures.

References

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Title of the talk

Zygmund's problem and axis parallel rectangles

Abstract

We will discuss a conjecture of Zygmund concerning maximal operators defined on a family of axis parallel rectangles in the Euclidean space. If the historical version of the problem has been disproved by Soria, we will see that the idea behind Zygmund's conjecture may still be true.

In particular, a certain reformulation of the problem has been solved in the Euclidean plane by Stokolos but it remains open in higher dimensions. In the past fews years, different authors (among whom D'Aniello, Hagelstein, Oniani, Moonens, Rey, Stokolos etc.) have established sharp weak type estimates in specific settings and their work lends weight to a certain reformulation of Zygmund's conjecture.

We will discuss this problem and in particular, I would like to focus on a specific family of rectangles that exhibits a product structure.

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Title of the talk

Almost a Hilbert Space

Co-authors

Douglas Mupassiri, Erdal Gül and Daniel Williams

Abstract

Kuelbs [1] proved that every separable Banach space $\mathcal{B}$ can be densely and continuously embedded in a Hilbert space $\mathcal{H}$. Let $\mathcal{L}[\mathcal{B}]$ be the bounded linear operators on $\mathcal{B}$. In this talk I will prove the special concrete case of $\mathbb{C}[0,1] \subset L_2[0,1]$, for the following general results (see [2, 3]):

1. For each $u \in \mathcal{B}$, there exists a semi-inner product $[\,\cdot, u]_z$ generated by bounded linear functional $u_z^* \in \mathcal{B}^*$ and a constant c_u such that $u_z^*(v) = [v,u]_z = c_u(v,u)_{\mathcal{H}}$ for every $v \in \mathcal{B}$.
2. If $\mathbb{A} \neq \mathcal{B}$ is closed, there exists $\mathbb{A}^{\perp} \subset \mathcal{B}$ disjoint, and $\mathcal{B} = \mathbb{A} \oplus \mathbb{A}^{\perp}$.
3. For $A \in \mathcal{L}[\mathcal{B}]$ there exists $A^* \in \mathcal{L}[\mathcal{B}]$ and $(A^*A)^* = A^*A$.
4. $\mathcal{L}[\mathcal{B}] \subset \mathcal{L}[\mathcal{H}]$ as a continuous dense embedding.
5. $\mathcal{B} \times \mathcal{B}^*$ has an Auerbach basis.
6. Every compact operator on $\mathcal{B}$ is the limit of operators of finite rank without assuming that $\mathcal{B}$ has a Schauder basis.
7. The Schatten class $\mathbb{S}_p[\mathcal{B}]$ exists for each $p \in [1,\infty]$, $\mathbb{S}_p[\mathcal{B}] \subset \mathbb{S}_p[\mathcal{H}]$ as a continuous dense embedding, $\bar{A} \in \mathbb{S}_p[\mathcal{H}]$ if and only if its restriction $A \in \mathbb{S}_p[\mathcal{B}]$, and $\left\|\bar{A}\right\|_p^{\mathcal{H}} = \|A\|_p^{\mathcal{B}}$.

These results lead us to conclude that every separable Banach space is almost a Hilbert space. (see [3]).)

References

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Title of the talk

Recent Progress on the Halo Conjecture

Abstract

The *Halo Conjecture* is one of the outstanding open problems in the theory of differentiation of integrals. In this talk we will provide an overview of issues surrounding the halo conjecture. In particular, the characterization of translation invariant density bases via Tauberian constants due to Hagelstein and Parissis will be discussed, as well as the recent result of Hagelstein and Stokolos that any homothecy invariant density basis of convex sets in $\mathbb{R}^2$ necessarily differentiates $L^p(\mathbb{R}^2)$ for every $1 < p \leq \infty$.

References

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Title of the talk

Big and Little lip and Quasiconvex Spaces

Co-author

Estibalitz Durand Cartagena

Abstract

Given a metric space (X, d) and $f : X \to \mathbb{R}$ with $D_r f(x) = \sup_{d(x,y)\leq r} \frac{|f(x) - f(y)|}{r}$ the so called "Big Lip" and "Little Lip" functions are defined as follows:

$$\mathrm{Lip} f(x) = \limsup_{r\to 0^+} D_r f(x) \qquad \mathrm{lip} f(x) = \liminf_{r\to 0^+} D_r f(x) \ .$$

Then we define

$$D(X) = \{f : X \to \mathbb{R} \,|\, ||\mathrm{Lip} f(x)||_\infty < \infty\} \quad d(X) = \{f : X \to \mathbb{R} \,|\, ||\mathrm{lip} f(x)||_\infty < \infty$$

$$\mathrm{Lip}(X) = \{f : X \to \mathbb{R} \,|\, f \text{ is Lipschitz on } \mathbb{R}.\}.$$

Note that $\mathrm{Lip}(X) \subset D(X) \subset d(X)$.

The metric space (X, d) is called quasiconvex if there exists $K < \infty$ such that given any points $x, y \in X$ there exists a curve γ connecting x and y such that $l(\gamma) \leq K d(x, y)$, where $l(\gamma)$ is the length of the curve γ.

If X is quasiconvex, then $\mathrm{Lip}(X) = D(X) = d(X)$. This turns out to be straightforward to prove. Some interesting questions arise when exploring the converse relations, i.e. if $\mathrm{Lip}(X) = D(X)$ or $D(X) = d(X)$ what can we conclude about the convexity of X? This talk presents joint work in this area by the speaker and Estibalitz Durand Cartagena.

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Title of the talk

On generalized topology related to the positive upper density

Co-authors

Renata Wiertelak and Władysław Wilczyński

Abstract

The essence of the density topology is the family of Lebesgue measurable sets for which each point of the set is its density point. The motivation of this presentation is investigation the family of measurable sets such that at every point of the set belonging to this family the upper density of this set is positive. We obtain strong generalized topology which essentials properties are provided in the virtue of properties of the classical density topology.

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Title of the talk

Densities and ideals

Co-authors

Małgorzata Filipczak, Tomasz Filipczak

Abstract

The idea of defining some families of subsets of the set $\mathbb{N}$ of all positive integers, using a notion of "sparseness" near zero of subsets of the real line $\mathbb{R}$ comes from [5]. In that paper a nice connection between a classical notion of a right-hand dispersion point of a measurable subset of $\mathbb{R}$ and a notion of a subset of $\mathbb{N}$, having density zero is shown.

Therefore, having a "nice" notion of "sparseness" near zero, which can be described by some kind of density ([4]), of unions of intervals on the real line $\mathbb{R}$ we can define ideals of subsets of $\mathbb{N}$. It seems to be interesting to verify when this method leads us to already known ideals. Following this idea "sparseness" in O'Malley sense have been associated with summable ideals ([3]). We consider an analogous relationship between f-density on $\mathbb{R}$ ([2]) and ideals of simple density on $\mathbb{N}$ defined in [1]. We also explore a relationship beetween these two kinds of ideals - the summable ideals associated with O'Malley density and the ideals produced by f-density.

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Title of the talk

A General Geometric Generator Filling Space

Abstract

In this paper we are concerned with space-filling curves that map a $[0,1]$ (or more generally, a linear perfect set) onto a closed domain, $D \subset \mathbb{R}^n$. For each such function there is a perfect core, $K \subset [0,1]$ with $f(K) = D$ such that K is minimal in this respect. We'll give an example or two using classical curves, show such cores can be far from unique for a given f and give a characterization for when a perfect set is a core for a given f.

Too, any space-filling curve, when restricted to a core can be arithmetically analyzed using a nested sequence of linearly ordered partitions of the target domain, D and reciprocally, any such nested sequence of linearly ordered partitions unequally generates a corresponding space-filling curve.

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Title of the talk

On some modifications of Darboux property

Co-authors

Elżbieta Wagner-Bojakowska, Aleksandra Karasińska

Abstract

In [1] A. Maliszewski modified the Darboux property and introduced so-called strong Świątkowski property. A function $f : \mathbb{R} \to \mathbb{R}$ has the strong Świątkowski property if for each interval $(a, b) \subset \mathbb{R}$ and for each $\lambda \in < f(a), f(b) >$ there exists a point $x_0 \in (a, b)$ such that $f(x_0) = \lambda$ and f is continuous at x_0.

We introduce some families of functions $f : \mathbb{R} \to \mathbb{R}$ having the $\mathcal{A}$-Darboux property modifying the Darboux property analogously as it was done by Maliszewski replacing continuity with so-called $\mathcal{A}$-continuity, i.e., the continuity with respect to family $\mathcal{A}$ of subsets in the domain.

We prove, among others, that if $\mathcal{A}$ satisfies certain conditions, then the family of functions having the $\mathcal{A}$-Darboux property is contained and dense in the family of Darboux quasi-continuous functions.

We prove also that for some families $\mathcal{A}$ the family of all Darboux $\mathcal{A}$-continuous functions is strongly porous in the space of all functions having the $\mathcal{A}$-Darboux property with the supremum metric.

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Title of the talk

On composition of Baire functions

Abstract

We discuss the maps between topological spaces whose composition with Baire class α maps also belongs to the α'th Baire class. We give characterizations of such maps and show that under some restrictions on the domain and the range the property of being the right Baire-one compositor is equivalent to many other function properties such as piecewise continuity, G_δ-measurability, B_1-stability, while left compositors are exactly continuous maps. By definition, for an ordinal $\alpha \in [0, \omega_1)$ a map $f : X \to Y$ between topological spaces is *the right (left)* B_α*-compositor for a class* $\mathcal{C}$ *of topological spaces* if for any topological space $Z \in \mathcal{C}$ and a map $g : Y \to Z$ (respectively, $g : Z \to X$) of the α'th Baire class the composition $g \circ f : X \to Z$ (respectively, $f \circ g : Z \to Y$) also belongs to the α'th Baire class. Such maps for $X = Y = \mathbb{R}$, $\mathcal{C} = \{\mathbb{R}\}$ and $\alpha = 1$ were introduced and studied by Dongsheng Zhao in [1].

The following two theorems are the main results.

Theorem 1. *Let (X, d_X) be a metric space, (Y, d_Y) be a metric space and $f : X \to Y$ be a map. Consider the following conditions:*

1. *f is of the first stable Baire class (i.e., there exists a sequence of continuous maps $f_n : X \to Y$ such that for every $x \in X$ there is $k \in \mathbb{N}$ with $f_n(x) = f(x)$ for all $k \geq n$);*

2. *f is piecewise continuous (i.e., there exists a countable closed cover $\mathcal{F}$ of X such that $f|_F$ is continuous for every $F \in \mathcal{F}$);*

3. *for any function $\varepsilon : Y \to \mathbb{R}^+$ there exists a function $\delta : X \to \mathbb{R}^+$ such that for all $x, y \in X$*

$$d_X(x, y) < \min\{\delta(x), \delta(y)\} \implies d_Y(f(x), f(y)) < \min\{\varepsilon(f(x)), \varepsilon(f(y))\}; \tag{1}$$

4. *for any function $\varepsilon : Y \to \mathbb{R}^+$ of the first Baire class there exists a function $\delta : X \to \mathbb{R}^+$ such that (1) holds for all $x, y \in X$;*

5. *f is the right B_1-compositor for the class of all metrizable connected and locally path-connected spaces;*

6. *f is G_δ-measurable and σ-discrete (which means that there exists a family $\mathcal{B} = \bigcup\limits_{n=1}^{\infty} \mathcal{B}_n$ of subsets of X, which is called a base for f, such that for any open set $V \subseteq Y$ there is a subfamily $\mathcal{B}_V \subseteq \mathcal{B}$ with $f^{-1}(V) = \bigcup \mathcal{B}_V$ and each family $\mathcal{B}_n$ is discrete in X).*

Then (1)⇒(2)⇒(3)⇒(4)⇒(5)⇔(6). If X is a hereditarily Baire space, then (6)⇒(2). If, moreover, Y is a path-connected space and $Y \in \sigma\mathrm{AE}(X)$, then all the conditions (1)–(6) are equivalent.

Theorem 2. *Let X be a T_1-space, Y be a perfectly normal space, $f : X \to Y$ be a map and $\alpha \in [1, \omega_1)$. If one of the following conditions holds:*

(i) $\alpha = 1$ and X is a connected and locally path-connected metrizable space, or

(ii) $\alpha > 1$ and X is a first countable space such that for any finite sequence $U_1, \dots, U_n$ of open subsets of X there exists a continuous map $\varphi : [1, n] \to X$ with $\varphi(i) \in U_i$ for every $i \in \{1, n\}$,

then the following conditions are equivalent:

1. *f is continuous;*
2. *f is the left B_α-compositor.*

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Title of the talk

The completeness of the Lorentz space defined by nonlinear integrals

Abstract

Let $\mathcal{F}_0(X)$ denote the collection of all $\mathcal{A}$-measurable real-valued functions on a measurable space $(X,\mathcal{A})$. Let $0<p<\infty$ and $0<q\leq\infty$. For a σ-additive measure μ on $(X,\mathcal{A})$, the Lorentz quasi-seminorm and the Lorentz space are defined by

$$\|f\|_{p,q} := \begin{cases} \left(p\displaystyle\int_0^\infty \left[t\mu(\{|f|>t\})^{1/p}\right]^q \frac{dt}{t}\right)^{1/q} & \text{if } q<\infty, \\ \displaystyle\sup_{t>0} t\mu(\{|f|>t\})^{1/p} & \text{if } q=\infty \end{cases} \tag{1}$$

and

$$\mathcal{L}^{p,q}(\mu) := \{f\in\mathcal{F}_0(X)\colon \|f\|_{p,q}<\infty\}.$$

Then (1) can be expressed as

$$\|f\|_{p,q} = \begin{cases} \left(\dfrac{p}{q}\right)^{1/q} \mathrm{Ch}(\mu^{q/p},|f|^q)^{1/q} & \text{if } q<\infty, \\ \mathrm{Sh}(\mu^{1/p},|f|) & \text{if } q=\infty \end{cases} \tag{2}$$

by using the Choquet integral and the Shilkret integral

$$\mathrm{Ch}(\mu,|f|) := \int_0^\infty \mu(\{|f|>t\}dt, \quad \mathrm{Sh}(\mu,|f|) := \sup_{t>0} t\mu(|f|>t\}),$$

both of which are nonlinear integrals widely used in nonadditive measure theory. For this reason, the Lorentz space $\mathcal{L}^{p,q}(\mu)$ can be defined by (2) even when μ is nonadditive.

In discussing the completeness of the Lorentz space defined by nonlinear integrals, we face the following problems.

- The functional $\|\cdot\|_{p,q}$ generally satisfies no tractable inequalities such as the triangle inequality.
- The set functions $\mu^{q/p}$ and $\mu^{1/p}$ are not necessarily additive even if μ is additive.
- Some known proofs of the completeness of the Lorentz space use Cauchy's criterion concerning convergence in μ-measure of measurable functions. What characteristic should be imposed on the nonadditive measure μ for the validity of this criterion.
- What types of convergence theorems of the Choquet and Shilkret integrals are needed for the proof?

In this talk, we consider what kind of characteristics should be imposed on the nonadditive measure μ to solve the above problems. A part of this research is based on a joint work with Mr. Naoki Yamada (`https://doi.org/10.1016/j.fss.2022.10.001`).

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Title of the talk

On $\mathfrak{c}$-lineability and $\mathfrak{c}$-spaceability of families of functions

Co-author

Małgorzata Turowska

Abstract

The talk is devoted to the recent trend in mathematical analysis of the search for algebraic structures, such as linear spaces or algebras with large dimension, in "small" sets of functions. Given a certain property we say that the subset M of a vector space X which satisfies this property is κ-lineable if $M \cup \{0\}$ contains a vector space of dimension κ, where κ is a cardinal number. If X is a topological vector space then we say that $M \subset X$ is κ-spaceable if $M \cup \{0\}$ contains a **closed** vector space of dimension κ. Obviously, if M is κ-spaceable then it is κ-lineable. If M contains an infinite-dimensional (closed) vector space, it shall be simply called lineable (respectively, spaceable). These notions of lineability and spaceability were created by Gurariy and first introduced in [1, 5].

Origins of this theory of lineability and spaceability date back to 1966, when a famous example due to Gurariy ([3, 4]) who showed that there exists an infinite-dimensional linear space in which every non-zero element is a continuous nowhere differentiable function on $[0, 1]$. (This result was later strengthened in many ways.) Lately, many authors have become interested in this subject and a wide range of similar constructions were given.

We study $\mathfrak{c}$-lineability and $\mathfrak{c}$-spaceability of some families of functions. The main goal is to formulate general conditions under which any non-empty family $\mathcal{F}$ of functions is $\mathfrak{c}$-spaceable or $\mathfrak{c}$-lineable. We consider the families of function of the form $\mathcal{F} = \mathcal{F}_1 \setminus \mathcal{F}_2$. Most often, family $\mathcal{F}_2$ is seemingly "very close" to $\mathcal{F}_1$ or consists of "almost all" functions.

The main idea of our constrctions is to "reproduce" one function to obtain $\mathfrak{c}$-dimensional (closed) linear space. For this "reproduction" we use the Fichtenholz-Kantorovich Theorem, applied to a countable family of pairwise disjoint intervals contained in the domain of functions. The initial function is "squashed" and "pasted" into disjoint intervals included in the domain of constructed function. The obtained results are a generalization of previous ideas of Bartoszewicz, Filipczak and Terepeta, [2].

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Title of the talk

Some classes of topological spaces extending the class of Δ-spaces

Co-authors

Jerzy Kąkol, Arkady Leiderman

Abstract

A topological space X is said to be a Δ-*space* if for any sequence $A_1 \supset A_2 \supset \ldots$ of subsets of X with $\bigcap_{n=1}^{\infty} A_n = \emptyset$, there is a sequence $G_1 \supset G_2 \supset \ldots$ of open subsets of X with $A_n \subset G_n$ and $\bigcap_{n=1}^{\infty} G_n = \emptyset$. The Δ-subsets of $\mathbb{R}$ were thoroughly investigated in the past. In particular, the existence of an uncountable Δ-subset of $\mathbb{R}$ is independent of ZFC. In the setting of a general topological space, the notion of a Δ-space was first investigated by J. Kąkol and A. Leiderman.

We study several related properties of topological spaces, especially the following one. If we consider only countable sets A_n in the definition of a Δ-space, we obtain a larger class of spaces, let us call them Δ_1-*spaces*. We show that uncountable Δ_1-subsets of $\mathbb{R}$ exist in ZFC.

Further, a Čech-complete space X is a Δ_1-space if and only if it is scattered (i.e., any subset of X has an isolated point). If every point of a Hausdorff space X is a G_δ-point, then X is a Δ_1-space if and only if every countable subset of X is a G_δ-set.

The talk is based on a joint work with Jerzy Kąkol and Arkady Leiderman. For more details, see [2].

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Title of the talk

Recurrent subspaces in Banach spaces.

Abstract

This talk is based in the work [2]. We will discuss the *spaceability* of the set of recurrent vectors $\mathrm{Rec}(T)$ for an operator $T : X \longrightarrow X$ acting on a Banach space X. In particular: following [3] we will give sufficient conditions for an operator to have a recurrent subspace; and following [1] we will characterization the quasi-rigid operators admitting a recurrent subspace. As a consequence we obtain that: *a weakly-mixing operator on a real or complex separable Banach space has a hypercyclic subspace if and only if it has a recurrent subspace*. The results exposed exhibit a symmetry between the hypercyclic and recurrent spaceability theories showing that, at least for the *spaceable* property, hypercyclicity and recurrence can be treated as equals.

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Title of the talk

Convergence for varying measures

Co-authors

Luisa Di Piazza, Kazimierz Musiał, Anna Rita Sambucini

Abstract

Some limit theorems of the type

$$\int_\Omega f_n \, dm_n \to \int_\Omega f \, dm$$

are presented for scalar, (vector), (multi)-valued sequences of m_n-integrable functions f_n. In [1] sufficient conditions in order to obtain some kind of Vitali's convergence theorems for a sequence of (multi)functions $(f_n)_n$ integrable with respect to a sequence of measures $(m_n)_n$ are considered. In particular we consider the asymptotic properties of $(\int_\Omega f_n dm_n)_n$ with respect to setwise and in total variation convergences of the measures in an arbitrary measurable spaces $(\Omega, \mathcal{A})$.

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Title of the talk

Mixing and invariance criteria for linear operators on Banach spaces with respect to infinitely divisible measures

Co-author

Nicolas Privault

Abstract

Criteria for the mixing of bounded linear operators on Banach spaces with respect to invariant Gaussian measures have been obtained in Bayart and Grivaux [1], Bayart and Matheron [2, 3].

The aim of this talk is to extend those criteria to the mixing of operators under a wider class of infinitely divisible measures that includes stable measures which are a generalization of Gaussian measures. We also discuss the role of the σ-spanning criterion [1, 2, 3] in the derivation of sufficient conditions for the existence of invariant stable measures.

Our approach uses criteria for mixing obtained in Rosiński and Żak [4], Passeggeri and Veraart [5] in the setting of discrete-time stochastic processes, and the definition of a codifference operator that extends the concept of covariance encountered with Gaussian measures.

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Title of the talk

Henstock Kurzweil almost periodic functions and their applications to differential equations

Abstract

In this talk we investigate some properties of the normed space of almost periodic functions which are defined using the Henstock-Kurzweil integral. In particular, we prove that this space is barrelled while it is not complete. We also prove that a linear differential equation with the non-homogenous term being an almost periodic function of such type, possseses a solution in the class under consideration.

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Title of the talk

Backward shifts for nonseparable spaces

Co-author

A. Zagorodnyuk

Abstract

Let X be an infinite-dimensional (nonseparable) Banach space which admits a Schauder decomposition to Banach spaces X_k, $k = 0, 1, \ldots$, that is, every $x \in X$ can be uniquely represented as

$$x = \sum_{k=0}^{\infty} x_k, \quad x \in X_k,$$

and the series converges in X.

Let $(F_k)_{k=1}^{\infty}$ be a sequence of injective maps $F_k \colon X_{k+1} \to X_k$ with dense ranges and $\|F_k\| = 1$. So we have the following shifts of spaces under maps F_k:

$$0 \longleftarrow X_0 \xleftarrow{F_1} X_1 \xleftarrow{F_2} \cdots \xleftarrow{F_n} X_n \cdots .$$

Let us define backward shifts operator by

$$T(x) = \sum_{k=1}^{\infty} \lambda_k F_k(x_k), \tag{1}$$

$$T \colon (x_0, x_1, \ldots, x_n, \ldots) \mapsto (\lambda_1 x_1, \lambda_2 x_2, \ldots, \lambda_n x_n, \ldots),$$

where λ_k are positive numbers with $\sup_k \lambda_k < \infty$. Clearly that T is continuous.

Theorem 0.1. *Suppose that the following assumptions hold*

1. *The weight constants λ_k are such that*

$$\limsup_{n\to\infty} \prod_{k=1}^{n} \lambda_k = \infty.$$

2. *There is a dense subspace* $E_0 \subset \operatorname{range}(F_1) \subset X_0$ *such that for every* $x \in E_0$ *the sequence*

$$F_n^{-1} \circ \cdots \circ F_1^{-1}(x), \quad n \in \mathbb{N}$$

is bounded in X.

Then the operator T *defined by* (1) *is topologically transitive.*

Note that condition (2) in Theorem 0.1 is evidently true if F_k are isomorphisms. Such transitive operators for the case of Hilbert spaces were considered in [1].

Corollary 0.2. *If a Banach space do not admits a topologically transitive operator then it can not be represented as a countable Schauder decomposition to isomorphic Banach spaces.*

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Title of the talk

Spectre of a set and achievement sets of series in $\mathbb{R}^2$

Co-author

Mateusz Kula

Abstract

Let $(X, +)$ be an Abelian group. Let $A \subset X$. We define the spectre of a set A as

$$S(A) := \{x \in X \colon \forall_{y \in A} y + x \in A \text{ or } y - x \in A\}.$$

We will show some properties of the spectre of a set and its connections with the centre of distances and achievement sets of absolutely convergent series in $\mathbb{R}^2$.

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Title of the talk

Almost everywhere convergence of nets of operators and weak type maximal inequalities

Abstract

The weak type maximal principles of Stein and Sawyer are extended to nets of operators defined on classes of functions on general measure spaces (possibly of infinite measure), including the case of locally compact groups. Applications to differentiation of integrals, multiple Fourier series and multi-parameter ergodic averages are given.

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Title of the talk

Chaotic weighted shifts on directed trees

Co-author

Karl G. Grosse-Erdmann

Abstract

The problem of characterizing when a unilateral or a bilateral weighted backward shift is chaotic has been completely solved by Grosse-Erdmann. We will discuss the generalization of this problem for weighted backward shifts on directed trees. Specifically, we will characterize when such operators are chaotic when acting on general Fréchet sequence spaces defined on either a rooted or unrooted directed tree. When the underlying space is of type ℓ^p, $1 \leq p < \infty$ or c_0, the characterizations can be expressed via generalized continued fractions which depend on the weight family and the geometry of the tree.

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Title of the talk

Reflexivity and weak sequential completeness in spaces of ideal operators and homogeneous polynomials

Abstract

In the year 1927 the famous Austrian mathematician Hans Hahn $(1879 - 1934)$ introduced the concept of reflexive normed space. Since then many mathematicians have been attracted to its properties, among them we find the works of Billy James Pettis, Shizuo Kakutani,William Frederick Eberlein,Witold Lwowitsch Šmulian y Robert C. James, among others.

The most important results that characterize reflective spaces are the following:

Teorema 0.1. *(Pettis, 1938) Let be X a reflexive Banach space, then X is reflexive if only if X' is reflexive.*

Teorema 0.2. *(Kakutani (Conway, 1985, p.132)) If X is a Banach space, then X is reflexive if and only if the closed unit ball of X is compact in the weak topology.*

Teorema 0.3. *(Eberlein-Šmulian (Diestel, 1984)) If X un espacio de Banach, entonces X es reflexivo si y sólo si toda sucesión acotada en X tiene una subsucesión débilmente convergente.*

Teorema 0.4. *(Teorema de James (James, 1964)) Si X is a Banach space, then X is reflexive if and only if every continuous linear functional in X reaches its maximum in the closed unit ball of X.*

Another concept that is closely related to reflexivity is the sequentially weakly complete Banach space. It is well known that every reflexive Banach space is sequentially weakly complete, however, the converse is not always true. For example, it is known that ℓ'_∞ is a sequentially weakly complete space but it is not reflexive. Rosenthal (Rosenthal, 1974) established the following theorem that relates the two concepts.

Teorema 0.5. *If X is a sequentially weakly complete Banach space, then X is reflexive or contains a subspace isomorphic to ℓ_1.*

(Qingying Bu 2013) proved the following theorem relating the reflexivity property and weakly complete sequentiality in the space of compact operators.

Teorema 0.6. *(Bu, 2013) Let X and Y be reflexive Banach spaces, then $\mathcal{K}(X;Y)$ is reflexive if and only if it is sequentially weakly complete.*

In this talk we will analyze the previously mentioned theorem for **operator ideals** $\mathcal{I}$, which are more general than compact operator spaces $\mathcal{K}$.

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Title of the talk

Cancellation conditions on localizable Hardy spaces

Co-authors

Galia Dafni, Chun Ho Lau and Claudio Vasconcelos

Abstract

In this talk we discuss cancellation conditions on localizable hardy spaces $h^p(\mathbb{R}^n)$ for $0 < p \leq 1$. As application, we show a Hardy type inequality for theses spaces

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Title of the talk

Extremal mappings among nonexpansive mappings

Co-authors

Christian Bargetz, Michael Dymond

Abstract

We consider the space

$$\mathcal{M} := \{f\colon C \to C\colon\ \operatorname{Lip} f \le 1\},$$

of nonexpansive mappings on a closed and convex subset C of the Banach space X equipped with the metric of uniform convergence. We say that a mapping $f \in \mathcal{M}$ is *extremal* if it does not admit a representation as a nontrivial convex combination of two elements of $\mathcal{M}$.

We investigate such extremal mappings. We show for example that if C is the closed unit ball of a Banach space, then under some additional restrictions every isometry is extremal, in some other cases, however, not all isometries can be extremal. The talk is based on an ongoing joint research with C. Bargetz and M. Dymond.

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Title of the talk

A family of non-multigeometric Cantorvals

Co-authors

Jacek Marchwicki, Piotr Nowakowski

Abstract

A new family of achievable sets that are Cantorvals will be presented. Unlike most of known so far examples of Cantorvals, the new sets are not generated by multigeometric series. Some sets of the family provide examples of a Cantorval that added - algebraically - to itself any finite number of times remains a Cantorval. Moreover, all sets in the new family have the Lebesgue measure equal to the measure of their interior which is a phenomenon that might be true for all achievable Cantorvals, but that remains an open problem for at least six years now.

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Title of the talk

An alternate construcion of approximate phases for a theorem of Mochizuki about the Limiting Absorption Principle.

Abstract

In Theorems 4.1 and 5.1 of his monograph, [3], Mochizuki established the principle of limiting absorption for a class of Schrodinger operators of the form, $H = -Delta + V$, where $Delta$ is the Laplacian and V is a potential. More spesifically, he introduced a class of potentials and a class of intervals and showed that for potentials of his class the Principal of Limiting Absorption holds for the parts of the corresponding Schrodinger opertors over each of his intervals. His class of potentials is interersting inasmuch it contains contains the famous Wigner von-Neumann potetial. His class of intervals is also interesting inasmuch as it generalizes the class of intervals of the two 1978 Mochizuki-Uchiyama papers, [2], [4].

The proofs of his Theorems 4.1 and 5.1 have two ingredients. The first one is the concept of the Aproximate Phase is given by the definitions (K.71) and (K.7.2) of his monograph. The other one is Theorem 5.1 itself. (See also the JR Theorem in Section 4.4)

The purpose of the present talk is to give an alternate construction of Approximate phase for Schrodinger operators with Wigner von-Neumann potentials. This alternate constrauction is based on the asymptotic theory of the adiabatic oscillator of the Ben-Artzi and Devinatz paper [1]. Ben-Artzi and Devinatz did emphasize, that their work, in turn, was based on the work of Harris and Lutz, we refer to that circle of works as the asymptotics of the HaLuBeDe initial value problem.

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Title of the talk

Change of Variable for the Riemann Integral on the Real Line

Abstract

We present a very elementary version regarding the *Change of Variable Theorem for the Riemann Integral on the Real Line*, that is stronger than those usually found in textbooks and that is not a particular case of the well known version of H. Kestelman. Two examples are given.

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Title of the talk

Convergence for varying measures in the topological case

Co-authors

Luisa Di Piazza, Valeria Marraffa, Kazimierz Musiał

Abstract

Convergence theorems for sequences of scalar, vector and multivalued Pettis integrable functions on a topological measure space Ω of the type

$$\lim_n \int_A f_n dm_n = \int_A f dm, \qquad \text{for every} A \in \mathcal{B}. \tag{1}$$

are described for varying measures which are vaguely convergent.
In a previous paper [1] we have examinated the problem when the varying measures converge setwisely in an arbitrary measurable space. This type of convergence is a powerful tool since it permits to obtain strong results, for example the Vitali-Hahn-Saks Theorem or a Dominated Convergence Theorem. But sometime in the applications it is difficult, at least technically, to prove that the sequence $(m_n(A))_n$ converges to $m(A)$ for every measurable set A, unless e.g. the sequence $(m_n)_n$ is decreasing or increasing. So other types of convergence are examined, based on the structure of the topological space Ω. The results that will be showed are contained in [2].

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Participant

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Title of the talk

Some geometric and proximinality properties of the unit balls and unit spheres of Banach spaces.

Abstract

In 1986, Megginson characterized the mid-point local uniform convexity which is a geometric property of a Banach space in terms of a proximinality property, called approximative compactness, of the unit ball. In this talk, we will see that the well known geometric properties of Banach spaces such as uniform convexity, local uniform convexity and weakly local uniform convexity can also be characterized in terms of different proximinality properties of the unit spheres. It is natural to ask whether certain known proximinality properties of the unit balls or unit spheres can be characterized in terms of some geometric properties of Banach spaces. Partial answers to the question mentioned above will be discussed in this talk.

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Title of the talk

Differentiation of integrals by basis of rectangles

Abstract

It is well-known that the basis of rectangles with sides parallel to the coordinate axes differentiates integrals of functions from $L^p, p > 1$; however it does not for $p = 1$. Moreover, for the basis of arbitrarily oriented rectangles the differentiation fails even for L^∞.

A. Zygmund asked the question of whether the differentiation property for a summable function can be improved by choosing rectangles in a special direction. A number of related results were proven, the first one by J. M. Marstrand [1] with further specifications and generalizations by A. Nagel, E. Stein, and S. Wainger [2], P. Sjögren and P. Sjölin, [3], G. G. Oniani [4], G. A. Karagulyan [5], L. Moonens, [6] and some others.

In my talk differentiation of integrals of functions from the class $Lip(1,1)(I^2)$ with respect to the basis of arbitrarily oriented rectangles will be presented. The sharpness of the result, as well as the estimate of the rate of differentiation, will be discussed.

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Title of the talk

Recent results on HK_r-integration

Co-authors

Paul Musial, Valentin A. Skvortsov, Francesco Tulone

Abstract

Some recent results concerning HK_r-integration, in particular, characterizations of indefinite integrals and the relation between the HK_r- and approximate Henstock integrals, will be presented.

References

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Participant

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Title of the talk

On topologies generated by lower porosity

Co-author

Stanisław Kowalczyk

Abstract

Porosity of a set, defined in [2], is the notion of smallness more restrictive than nowhere density and meagerness. It can be defined in arbitrary metric space. The main idea is that we modify the "ball" definition of nowhere density by the request that the sizes of holes should be estimated. Usually, the notion of the (upper) porosity of sets is used in many aspects, see for example [2, 3, 4, 6, 8, 9]. We deal with the lower porosity, which also be considered in some papers, [7, 8]. It is known that there are big differences between the lower and the upper porosities. In [8, 9] some properties of the lower porosity in metric spaces are presented, whereas in [7] some properties of the lower porosity on $\mathbb{R}^2$ and of lower porouscontinuous functions $f\colon \mathbb{R}^2 \to \mathbb{R}$ are studied.

In [9] and [5] L. Zajíček and V. Kelar introduce two topologies using the notion of (upper) porosity and (upper) strong porosity. Let $A \subset X$ and $x \in X$. We say that A is (upper) superporous at x if $A \cup B$ is (upper) porous at x whenever B is (upper) porous at x. A set A is said to be p-open (porosity open) if $X \setminus A$ is (upper) superporous at any point of A.

We say that A is (upper) strongly superporous at x if $A \cup B$ is (upper) porous at x whenever B is (upper) strongly porous at x. A set A is said to be s-open (strongly porosity open) if $X \setminus A$ is (upper) strongly superporous at any point of A.

The system of all p-open sets in $(X, \| \|)$ forms a topology $p(X, \| \|)$, which will also be called the p-topology or the porosity topology, [9]. The system of all s-open sets forms a topology $s(X, \| \|)$, which will be called s-topology or the strong porosity topology, [5]. Obviously $p(X, \| \|)$ and $s(X, \| \|)$ are finer than the initial topology. On a non-trivial normed space neither $s(X, \| \|)$ is finer than $p(X, \| \|)$ nor $p(X, \| \|)$ is finer than $s(X, \| \|)$. The both topologies are completely regular, [5].

The aim of our talk is to describe the properties of topologies $\underline{s}(X, \| \|)$ and $\underline{p}(X, \| \|)$ which are generated by the lower porosity in a similar way as $s(X, \| \|)$ and $p(X, \| \|)$ were generated by the standard (upper) porosity.

We describe relationships between topologies $s(X, \| \|)$, $p(X, \| \|)$, $\underline{s}(X, \| \|)$, $\underline{p}(X, \| \|)$ and $\mathcal{T}_{\| \|}$, which are nontrivial.

The last part of the talk presents some applications of topologies $\underline{s}(X, \| \|)$ and $\underline{p}(X, \| \|)$. Namely, we define lower porouscontinuous functions, following ideas of J. Borsík and J. Holos from [1], and we describe maximal additive classes for some types of lower porouscontinuity in terms of topologies $\underline{s}(X, \| \|)$ and $\underline{p}(X, \| \|)$.

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Title of the talk

Stability analysis of a Lotka-Volterra type model by different techniques

Co-author

Delfim F. M. Torres

Abstract

We consider a modified Lotka–Volterra model with Michaelis–Menten type funcional response applied to the bank system, but can also be applied to other areas. We prove the model is well posed (non-negativity and boundedness of the solutions) and study the local stability using different methods. Firstly we consider the continuous model, after we investigate the dynamical consistency of two numerical schemes, namely, Euler and Mickens. Finally, the model is described using Caputo fractional derivatives. For the fractional model, besides well-posedness and local stability, we prove the existence and uniqueness of non-negative solutions. Throughout the work we compare the results graphically and present our conclusions. To represent graphically the solutions of the fractional model we use the modified trapezoidal method that involves the modified Euler method.

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Title of the talk

On sets intersecting every square of the plane

Co-author

Michael Hrušák

Abstract

In this talk we present some properties of sets that contain at least one vertex of each square of the plane, in particular we study minimal elements (with respect to the subset relation) of the family $\mathcal{A}$ of sets with this property. We will discuss this properties in the context of the square peg problem: Does every Jordan curve cointains the four vertices of an euclidean square?

Participant

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Title of the talk

On null ideals of Hausdorff and packing measures

Abstract

The *uniformity* of an ideal $\mathcal{J}$ of subsets of a set X is the least cardinality of a set $A \subseteq X$ such that $A \notin \mathcal{J}$. Other common cardinal invariants of an ideal are the *covering*, *additivity* and *cofinality*.

The cardinal invariants of the ideals of sets of zero s-dimensional Hausdorff measure on the real line (or, more generally, an analytic metric space) were studied by Fremlin, Ostaszewski, Shelah, Steprans, Elekes and others. Fremlin established their position in the Cichoń's diagram. Shelah and Steprāns proved that the uniformity may depend on the dimension s of the Hausdorff measure and Elekes and Steprāns proved similar results for the covering. We provide a few more results about the four cardinal invariants of Hausdorff measures, in particular we improve some estimates and calculate the precise values of the uniformity and covering of Hausdorff measures in the Baer–Specker group $\mathbb{Z}^\omega$. We also establish a parallel theory of cardinal invariants of packing measures with surprisingly different outcomes.