THE BAIRE HIERARCHY, MULTIFRACTAL DECOMPOSITION SETS AND Π^0_γ-COMPLETENESS

Lars Olsen

University of St. Andrews, Scotland

e-mail: lo-andrews.ac.uk

The Baire hierarchy provides a natural classification of the "complexity" of subsets of a metric space. For a metric space X and an ordinal γ with $1 \leq \gamma < \omega_1$ (where ω_1 is the first uncountable cardinal), the Baire classes $\Sigma^0_\gamma(X) = \Sigma^0_\gamma$ and $\Pi^0_\gamma(X) = \Pi^0_\gamma$ are defined inductively by

$$\Sigma^0_1(X) = \left\{ G \subseteq X \,\middle|\, G \text{ is open} \right\}, \quad \Pi^0_1(X) = \left\{ F \subseteq X \,\middle|\, F \text{ is closed} \right\},$$

and

$$\Sigma^0_\gamma(X) = \left\{ \bigcup_{n=1}^{\infty} E_n \,\middle|\, E_n \in \bigcup_{\kappa<\gamma} \Pi^0_\kappa(X) \right\}, \quad \Pi^0_\gamma(X) = \left\{ \bigcap_{n=1}^{\infty} E_n \,\middle|\, E_n \in \bigcup_{\kappa<\gamma} \Sigma^0_\kappa(X) \right\}.$$

We have the following inclusions

$$\begin{array}{ccccccc} \Sigma^0_1(X) & \subseteq & \Sigma^0_2(X) & \subseteq & \Sigma^0_3(X) & \subseteq & \ldots \\ \Pi^0_1(X) & \subseteq & \Pi^0_2(X) & \subseteq & \Pi^0_3(X) & \subseteq & \ldots \end{array} \quad ;$$

this list of inclusions is known as the Baire Hierarchy and gives a stratification of the Borel subsets of X in (at least) ω_1 levels.

This talk will discuss the position of the so-called "multifractal decomposition sets" in the Baire Hierarchy. In particular, we will prove that "multifractal decomposition sets" are the building blocks from which all other Π^0_γ sets can be constructed; more, precisely, "multifractal decomposition sets" are Π^0_γ-complete.

As an application we find the position of the classical Eggleston-Besicovitch set in the Baire Hierarchy.